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Development Research Framework for Designing Functions Class Using Desmos

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Abstract: Technology-enhanced learning aids are playing a more and bigger role in the constantly changing world of mathematics education. This paper introduced a comprehensive research framework aiming to revolutionise the design of functions classes through innovative Desmos application. The main goal was to develop research framework for Functions classes utilising Desmos as a tool. To actively involve students in the learning process of the framework, creative approaches for incorporating this technology tool into the curriculum were offered. Its interactive and dynamic characteristics were specifically highlighted. The framework was built on the ideas of design-based research and included iterative planning, design, implementation, and evaluation procedures. In the given investigation, the challenges faced by educators in teaching Functions, including the need for increased student engagement and the difficulty in visualising abstract mathematical concepts were uncovered. Significant findings from this study reveal that the integration of Desmos into Functions classes significantly improves student engagement, fosters a deeper understanding of mathematical concepts, and promotes problem-solving skills. Moreover, educators reported increased enthusiasm among students and a more enjoyable learning experience. The implications of this study are profound for the future of mathematics education. By harnessing the potential of Desmos, educators can design dynamic, captivating, and

productive functions classes that align closely with curriculum objectives. Our findings demonstrated that Desmos can be a catalyst for more engaging and effective mathematics instruction, thereby shaping a brighter future for students and educators alike.

Keywords: Desmos, Development Research Framework, Functions, Technology-Enhanced Learning

Introduction

Technology has become integral to teaching and learning in mathematics education. The use of technology in mathematics classrooms can enhance students' learning experiences by providing dynamic and interactive environments for exploring mathematical concepts and visualising graphs of functions. Desmos is an online graphing calculator and math learning platform that has recently gained popularity as a tool for teaching and learning mathematics. It is free, accessible, and offers a range of features and tools that can help students to explore and understand mathematical concepts. Despite the potential benefits of using Desmos in mathematics classrooms, research-based frameworks and guidelines must be developed for designing function classes using Desmos. Many teachers need to be fully aware of the capabilities of Desmos and may need to effectively gain the necessary skills to use it. In addition, integrating technology in mathematics classrooms can be challenging, and there is a need for research-based frameworks that can guide mathematics educators in designing and implementing effective mathematics classes using Desmos. One of the mathematics concepts the students need help with is function. The significance of this research lies in its response to several critical issues within the domain of mathematics education: (1) Engagement and Visualisation: Teaching functions has often been plagued by the challenge of engaging students and helping them visualise abstract mathematical concepts. Recent educational research has underscored the importance of active engagement and visualisation in promoting deep mathematical understanding (Svitek et al., 2022; Tokanov et al., 2023); (2) Technology Integration: With advancements in technology, there is a growing need to effectively integrate digital tools into the mathematics curriculum. However, the integration should not be superficial but rather transformative, aiming to enhance the learning experience and outcomes (Long & Bouck, 2022); (3) Design-Based Research: Contemporary educational research increasingly emphasises the importance of design-based research methodologies that allow for iterative development, implementation, and evaluation of innovative teaching approaches (Lim & Nguyen, 2022)

Research Problem

In the ever-evolving landscape of mathematics education, one critical challenge has persisted over the years - how to make learning functions an engaging, comprehensible, and productive experience for students (Gavin et al., 2009; Lundin, 2012). The abstract introduces a research problem of paramount importance, which revolves around the need to transform the conventional approach to teaching functions by harnessing the potential of technology-enhanced learning tools, specifically the Desmos application. Traditional instruction of functions has often been marred by abstract concepts that students struggle to grasp, resulting in disengagement and hindered learning outcomes. Teachers grapple with the dual challenge of cultivating student interest and bridging the gap between mathematical theory and practical understanding (Thompson, 1984; Chamberlin & Powers, 2010; Park et al., 2016). This research problem recognises these hurdles and seeks to address them through a well-structured development research framework.

Besides, some of the difficulties of the students understanding the concept of a function is that many of them need help understanding the abstract concept of a function, which is the relationship between two variables where each input has only one output. In their work, they found that students often need help understanding the concept of a function and identifying the independent and dependent variables. In addition to this, students also struggle with graphing functions accurately and interpreting the graphs. Students may need help graphing functions accurately and understanding the relationship between the graph and the function. Besides that students have difficulty manipulating and solving functions algebraically, including factoring, solving equations, finding inverse functions, and in the end, the students struggle to connect the algebraic and graphical representations of functions as well as the verbal representations of functions. Applying functions to real-world problems also becomes a problem for students when they need help connecting and understanding the practical applications of functions. In response to these challenges, our research framework adopts a design-based research approach, informed by the most recent scholarship in mathematics education and educational technology. By systematically designing, implementing, and evaluating functions classes that leverage the full potential of Desmos, the aim was to provide educators with a roadmap for the development of dynamic, interactive, and effective teaching and learning experiences.

The crux of the problem lies in reimagining mathematics instruction. It is essential to find innovative ways to integrate Desmos as a dynamic, interactive tool into the curriculum, making the learning of functions not just effective but also enjoyable. The research problem emphasises the need to design and implement a comprehensive framework based on design-based research principles that can lead to a reinvigorated, student-centric approach to teaching functions. Ultimately, this research problem serves as a call to action, highlighting the imperative to bridge the gap between conventional teaching methods and the digital era's potential. It seeks to provide a solution that significantly improves students' learning outcomes in functions classes, setting a new standard for mathematics education in the future.

Research Focus

Based on the several mentioned issues, this study explored possibilities framework to design functions classes using Desmos. The primary focus of this research was to investigate how Desmos, as a dynamic online graphing calculator, can be strategically integrated into the design and delivery of functions classes to address the pressing challenges in contemporary mathematics education. The developed framework can be grounded in educational research and best practices and adaptable to different contexts. The research framework will help mathematics educators to design and implement functions classes that engage students in meaningful learning experiences and support the development of their mathematical understanding.

Research Aim and Research Questions

This paper aimed to develop research framework for Functions classes utilising Desmos as a tool. The framework contained iterative processes for planning, design, implementation, and evaluation and was founded on the concepts of design-based research. The following research question to address the aim is "How can a development research framework leveraging Desmos as a technology-enhanced learning tool be designed and implemented to address the multifaceted challenges students face in understanding and learning functions in mathematics education?"

Expected outcomes based on the aim of this paper were:

1. A Comprehensive Research Framework: the research was expected to produce a well-structured development research framework that could serve as a model for mathematics educators to design and implement functions classes effectively using Desmos;

2. Improved Learning Outcomes: the implementation of the framework was expected to lead to improved learning outcomes, including increased student engagement, deeper understanding of mathematical concepts, and better problem-solving skills in functions;
3. Recommendations for Effective Integration: the research was expected to provide actionable recommendations for educators on how to integrate Desmos into functions classes to maximise its potential for enhancing teaching and learning;
4. Insightful Analysis: the research will yield insightful analysis about the impact of technology-enhanced learning on mathematics education, specifically in the context of functions. It will shed light on how mathematics instruction can evolve in the future.

In addition to this, the broader impact of this research lies in its potential to reshape mathematics education. By addressing the challenges students face in learning functions and providing educators with a practical framework for leveraging technology, the research has the potential to:

1. To enhance mathematics education: the research can contribute to the ongoing improvement of mathematics education by offering innovative strategies for teaching functions, making it more engaging and accessible to a wider range of students;
2. To empower educators: educators will benefit from the research findings and recommendations, as they can better integrate technology tools like Desmos into their teaching practices to improve instruction;
3. A benefit for students: The research has the potential to positively impact students by making the learning of functions more enjoyable, comprehensible, and aligned with real-world applications;
4. To inform educational policy: Findings from this research can inform educational policies and curriculum development, encouraging the adoption of technology-enhanced learning tools to enhance mathematics education. In summary, this research seeks to create a transformative framework for teaching functions, with the potential to improve both student learning outcomes and the overall landscape of mathematics education.

Research Methodology

General Background

The scope of literature review for this methodology focused on three key areas:

1. Development Research Frameworks in Mathematics Education: existing development research frameworks in the field of mathematics education, with a specific focus on those that have been used to enhance the teaching and learning of mathematical concepts were investigated. Then one explored how these frameworks have been designed and implemented and assess their impact on student learning outcomes.
2. Technology-Enhanced Learning in Mathematics: the literature related to the integration of technology, such as Desmos, into mathematics education was examined. Examine studies and research that highlight the effectiveness of technology-enhanced learning tools in improving students' understanding of mathematical concepts, engagement, and overall learning experiences.
3. Challenges in Teaching and Learning Functions: the existing body of literature that discusses the common challenges students face when learning functions in mathematics; include research on conceptual difficulties, graphing challenges, algebraic manipulations, and the integration of algebraic, graphical, and verbal representations of functions were explored.

When we indexing the literature for the methodology section, we organize it into subsections that align with the scope areas mentioned above:

1. Development Research Frameworks in Mathematics Education:
    a. Define development research frameworks in the context of mathematics education.
    b. Review relevant studies that have used development research methodologies to improve mathematics instruction.
    c. Analyse the key components of successful development research frameworks.
    d. Discuss the role of design-based research principles in the development of effective frameworks.
    e. Identify gaps or limitations in the existing literature related to development research in mathematics education.
2. Technology-Enhanced Learning in Mathematics:
    a. Discuss the significance of technology-enhanced learning tools in mathematics education.
    b. Present evidence from the literature regarding the impact of technology, specifically Desmos, on mathematics instruction.
    c. Examine studies that highlight the benefits of technology-enhanced learning in terms of student engagement, understanding, and problem-solving skills.
    d. Explore best practices for integrating technology tools into mathematics curriculum.
    e. Identify any emerging trends or innovations in technology-enhanced mathematics education.
3. Challenges in Teaching and Learning Functions:
    a. Summarise the common difficulties students encounter when learning functions in mathematics.
    b. Reference studies that have investigated these challenges, including those related to conceptual understanding, graphing, algebraic manipulations, and connecting different representations of functions.
    c. Analyse the implications of these challenges for mathematics education and curriculum development.
    d. Identify areas where technology-enhanced learning tools like Desmos can potentially address these challenges.
    e. Discuss the need for a comprehensive development research framework to tackle these issues effectively.

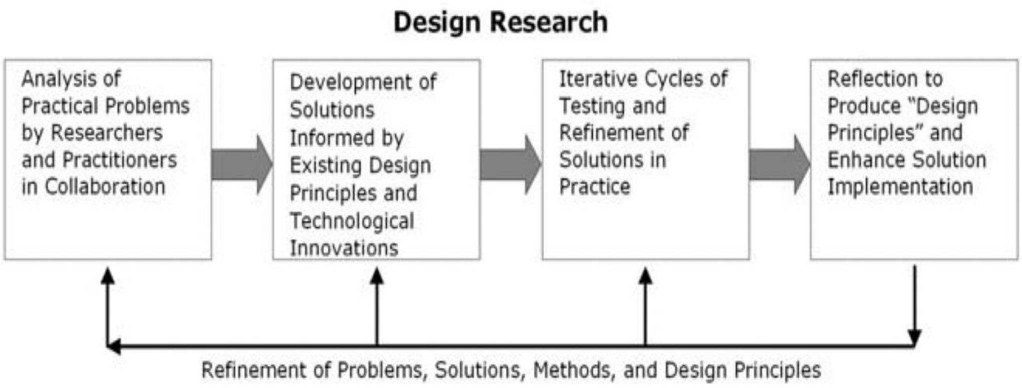
Based on the scope and indexing above, the integration of Desmos into the development of functions classes, aligning with the overarching goal of enhancing mathematics education comprehensively were investigated. By doing so, a design-based research proposed by Reeves (2006) was chosen. This facilitated the iterative planning, implementation, and evaluation. The research begun with an extensive literature review spanning the last decade to identify current trends, best practices, and scholarly insights related to the integration of technology, particularly Desmos, in mathematics education. This review informed the approach and ensures that our research builds upon and contributes to the existing body of knowledge (Lesh et al., 2000). A review of the literature on the use of technology in mathematics education (Reeves, 2006), an analysis of best practices in designing functions classes, and a study of the efficiency of using Desmos in functions classes were included in the research framework for designing functions classes using Desmos. Technology's potential to improve mathematics instruction and student learning outcomes was examined in the literature study. The benefits and difficulties of using technology in math courses were examined. Desmos was used to determine important characteristics of successful function classes and analyse best practices in function class design. Clear learning objectives, well-designed exercises, efficient scaffolding, and chances for students to participate in collaborative learning were some of these aspects. The framework was founded on the tenets of effective mathematics teaching (Goldin, 1998) and served as a manual for creating and implementing functional classes that engaged students in meaningful learning experiences. Then Goldin framework was used and modified based on this research aim addressing the challenges in teaching and learning functions.

Research Results

The framework contains iterative processes for planning, design, implementation, and evaluation and is founded on the concepts of design-based research. Design research approaches in educational technology steps proposed by Reeves (2006) reflect a dynamic and expanding field at the crossroads of instructional design, technological integration, and pedagogical innovation. This approach aims to bridge the gap between theory and practise by emphasising the systematic and iterative development of educational interventions, tools, and environments. At its foundation, design research in educational technology is guided by a dedication to tackling real-world educational difficulties (Molina et al., 2007; McKenney & Reeves, 2012). The design research differs from traditional research paradigms in that it prioritises the immediate relevance and impact of research findings. Design research approach often emphasise the significance of design principles and theoretical frameworks (see Fig. 1). Researchers use known theories of learning, cognition, and human-computer interaction to influence design decisions, ensuring that educational technology interventions are anchored in strong pedagogy. In essence, educational technology design research serves as a catalyst for educational innovation. It uses technology to produce more effective, interesting, and learner-centred educational experiences while also contributing to the larger discussion on the integration of technology in education. Design research seeks to impact the future of teaching and learning in an increasingly digital environment through rigorous design, collaboration, and continuing reflection.

Figure 1

Design research Approaches in Educational Technology



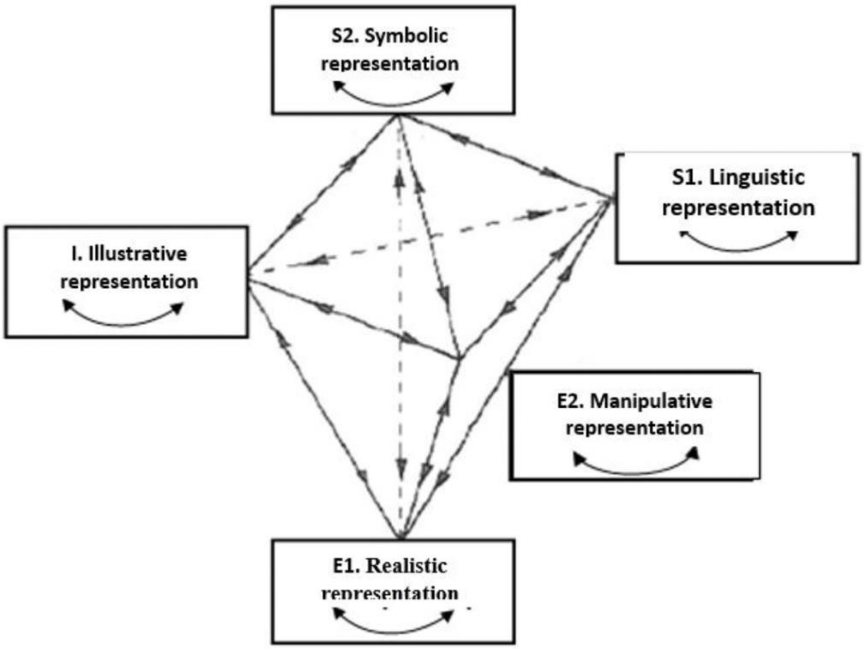
Source: Reeves (2006).

Works with it, to strengthen the framework, the main focus to overcome the issue of students' difficulty in learning is put emphasize on representation. Five modes of representation mentioned by Goldin (1998) are being used to explain the final proposed framework of this study: (1) Realistic representation is depictions of actual conditions and objects; (2) Manipulative representation: artificially created representations to enhance the dynamic activity of things; (3) Illustrative representation: representations that use illustrations, figures or graphs; (4) Linguistic representation:

representations in everyday languages, and the last one; (5) Symbolic representation: representations used in mathematical notations. Goldin's mode divides the enactive representation into two which are realistic representation and manipulative representation, while for the written symbol are distinct as linguistic and symbolic representation (Tan et al., 2023).

Figure 2

Goldin'*s Representational Systems in Mathematics' Education*



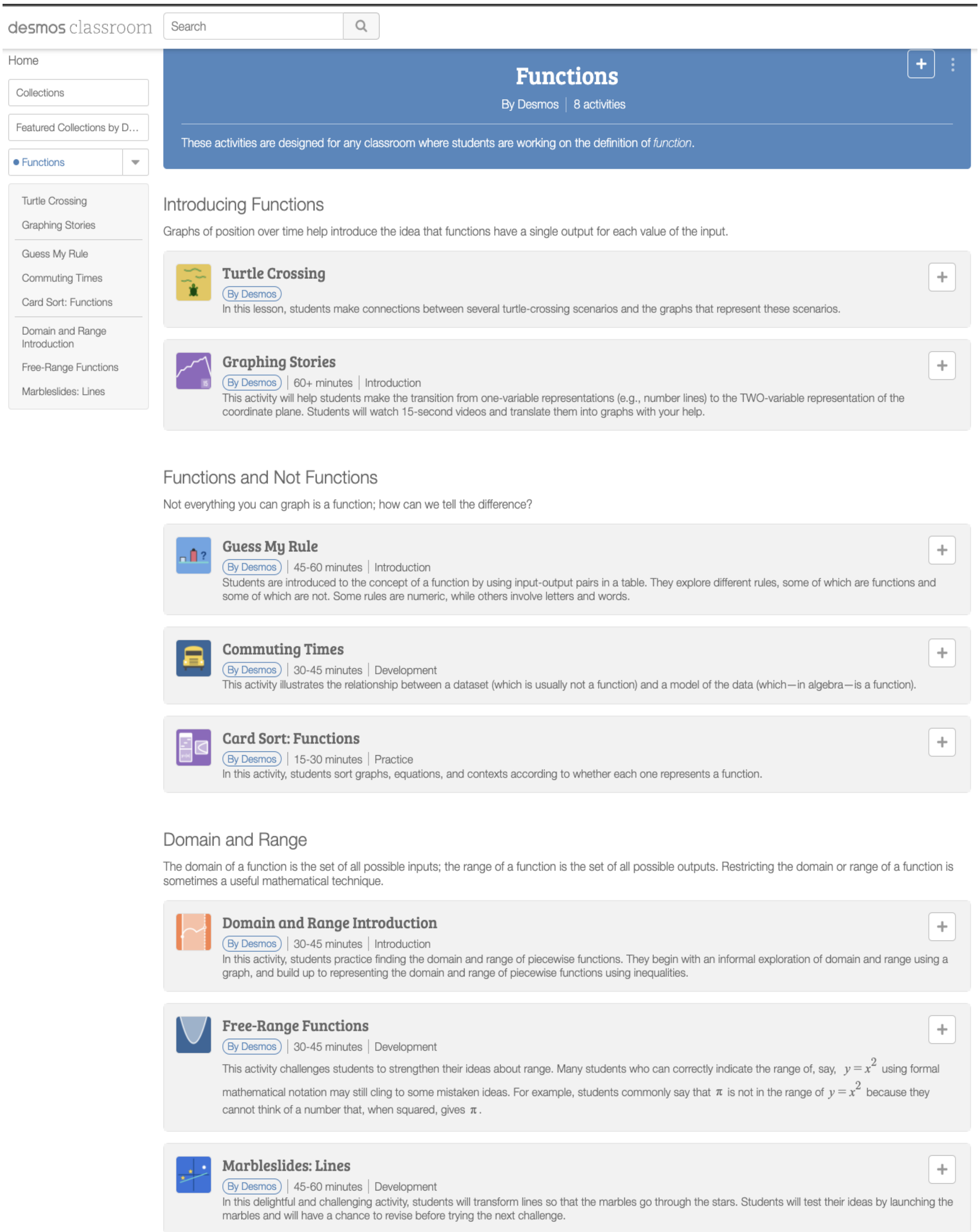
Source: Goldin (1998)

Planning

The planning stage holds immense significance as it lays the foundation for effective instruction. Reeves (2006), in his works on design research from the perspective of technology, emphasise collaboration between researchers and practitioners; in this sense, teachers prepare and explore the real teaching and problems given. In this study, to overcome the difficulties of function, Desmos is proposed. Eight activities (Figure 3) could be used to teach function (https://www.desmos.com/). The initial two activities which are introduce the function serve as the scaffolding upon which the entire lesson is constructed. Research in mathematics education has consistently emphasized the importance of establishing clear conceptual understanding before delving into procedural fluency (Hiebert & Grouws, 2017). These first two activities could be considered as building conceptual foundations. For the assessment of conceptual understanding, the subsequent three activities, which involve differentiating between functions and non-functions, serve a dual purpose. Firstly, they serve as formative assessments to gauge students' comprehension of the newly introduced concept. This aligns with the principles of assessment for learning, where ongoing assessment is used to inform instructional decisions (Black & Wiliam, 1998). Secondly, these activities foster critical thinking and metacognition, as students must analyze the characteristics of functions to distinguish them from other mathematical relationships. The final three activities, focusing on domain and range within functions, represent a natural progression in the instructional sequence. Once students grasp the fundamental concept of functions, they can explore the essential elements that define a function: its input values (domain) and output values (range). By dedicating a segment of the instruction to these aspects, we are not only deepening students' understanding but also aligning with research highlighting the significance of domain and range in function comprehension (Fries et al., 2020).

Figure 3

Desmos Activities for Function



Source: https://www.desmos.com/

To enhance the effectiveness of this instructional plan, it may be advantageous to include real-world applications of functions throughout the activities. Research indicates that contextualized mathematics instruction can improve students' motivation and transfer of learning (Lesh et al., 2000). This can be achieved by presenting functions in contexts that resonate with students' experiences, such as modelling real-world scenarios or exploring functions in fields like physics or economics.

While planning this sequence of activities, it is crucial to consider the diverse learning needs of students. Research on differentiated instruction suggests that adapting teaching methods to accommodate various learning styles and abilities can lead to improved student outcomes (Tomlinson, 2014). Thus, the planning stage should include strategies for differentiation, such as providing additional support for struggling learners and offering enrichment opportunities for those who grasp concepts quickly. In conclusion, the planning phase in mathematics instruction is a critical juncture that demands meticulous consideration. The proposed sequence of activities, starting with the introduction of the function concept and culminating in domain and range exploration, aligns with established principles of mathematics education and assessment. To enhance its effectiveness, integrating real-world contexts and incorporating differentiation strategies can further enrich the learning experience. Ultimately, a well-designed plan sets the stage for successful teaching and learning, ensuring that students develop a solid grasp of the concept of functions and its related components.

Design

Designing the classroom using Desmos in order to teach function provides students with an interactive and dynamic environment for exploring the concepts. It allows creating graphs of functions in real-time, making it easy for students to visualise the relationships between variables and understand the behaviour of functions. In the process of designing a classroom environment for teaching functions using Desmos, we embark on a pedagogical journey that combines technological innovation with sound instructional design principles. This stage of "design" is pivotal as it determines the structure and flow of the learning experience, aligning with the research findings that underscore the potential of Desmos to foster interactive and dynamic learning environments. The utilization of Desmos as a central tool in the classroom design transforms the traditional static learning environment into an interactive and responsive space. Research in educational technology highlights the importance of creating interactive learning environments that engage students actively in the learning process (Abykanova et al., 2016). By integrating Desmos an environment where students can actively explore and experiment with mathematical concepts, leading to deeper comprehension was created.

One of the defining features of Desmos is its capability to generate graphs of functions in real-time. This dynamic functionality aligns with the principles of constructivism, where learners actively build their understanding through hands-on exploration (Piaget, 1972). In the context of teaching functions, this means that students can instantly visualize the impact of different variables on function behaviour, fostering a more intuitive and experiential grasp of mathematical concepts. The design of the classroom with Desmos at its core is driven by the goal of enhancing students' conceptual understanding of functions. Research indicates that visualization and interactive tools can support students in developing a more profound and flexible understanding of mathematical concepts (Etyarisky, & Marsigit, 2022). By allowing students to see and manipulate graphs in real-time, Desmos facilitates the exploration of key function attributes, such as transformations, domain, and range, leading to richer conceptual development.

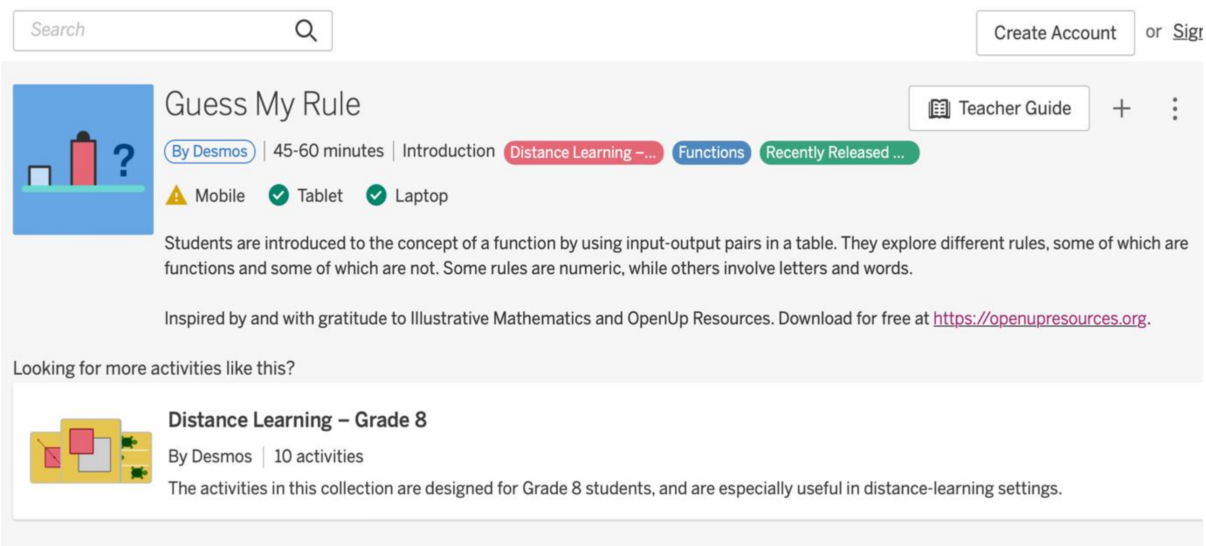
Incorporating Desmos into the classroom design also opens avenues for collaborative learning. Students can work together in real-time, sharing graphs, discussing observations, and jointly solving problems. Collaborative learning has been shown to enhance student engagement and promote deeper understanding through peer interaction (Johnson & Johnson, 2009). The classroom design encourages cooperative exploration of functions, further reinforcing learning outcomes. Rose & Meyer (2002) stated that an essential aspect of classroom design with Desmos is ensuring accessibility and inclusivity for all students. Research emphasises the importance of designing technology-enhanced learning environments that accommodate diverse learner needs. Therefore, the classroom design should consider accessibility features and provide support for students with varying abilities, ensuring that all learners can benefit from the interactive and dynamic environment. Desmos also provides a range of tools and features, such as sliders, tables, and data plots, that can help students to explore and manipulate mathematical concepts. When designing function classes using Desmos, it is important to define learning objectives and design tasks aligning with them. Tasks should be designed to promote the development of mathematical thinking and reasoning and should provide opportunities for students to engage in problem-solving and sense-making activities. Based on the eight activities in Desmos (Fig. 3), the activity that is most suitable for introducing functions is an activity entitled Guess My Rule. In conclusion, the design of a Desmos-integrated classroom for teaching functions is a deliberate and research-informed process. It leverages the power of technology to create an interactive and dynamic learning space that promotes deeper conceptual understanding, collaboration, and inclusivity. This design aligns with contemporary educational research, highlighting the transformative potential of technology in enhancing mathematics education. By designing classrooms that embrace Desmos, we pave the way for a more engaging and effective learning experience for students as they explore the intricacies of mathematical functions.

Implementation

Through the Guess My Rule activities, Desmos analogises the basic concept of function to an animation illustration of a working machine (Desmos.com., n. d). Guess My Rule in Desmos has eleven stages within 60 minutes, where the students are required to explore the rule to determine whether it is a function.

Figure 4

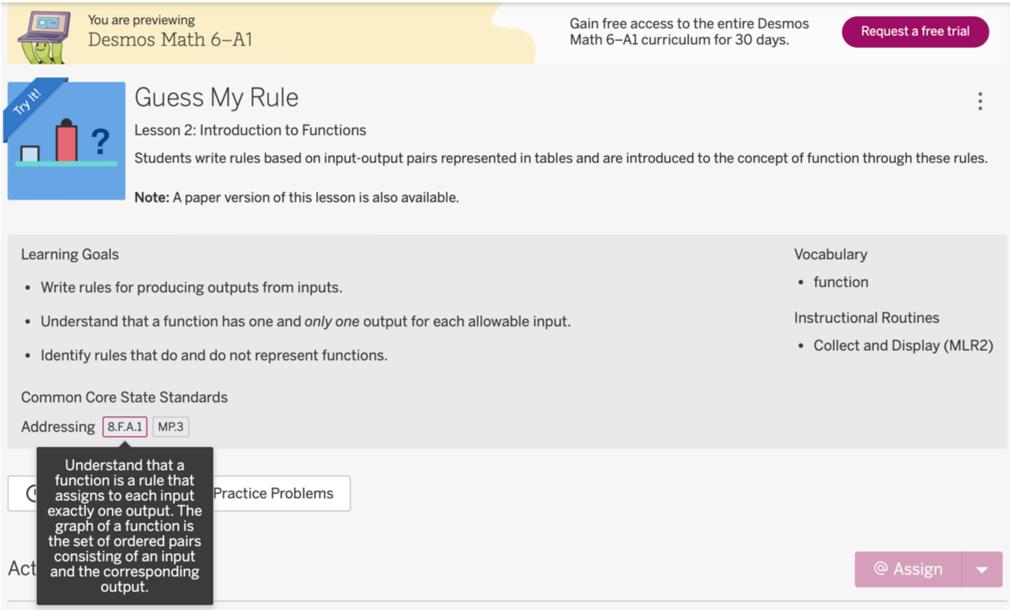
Desmos Activities for Function Guess my Rule in Teacher Guide Page



Source: https://www.desmos.com/

Figure 5

Desmos Activities for Function Guess my Rule in Students' Page

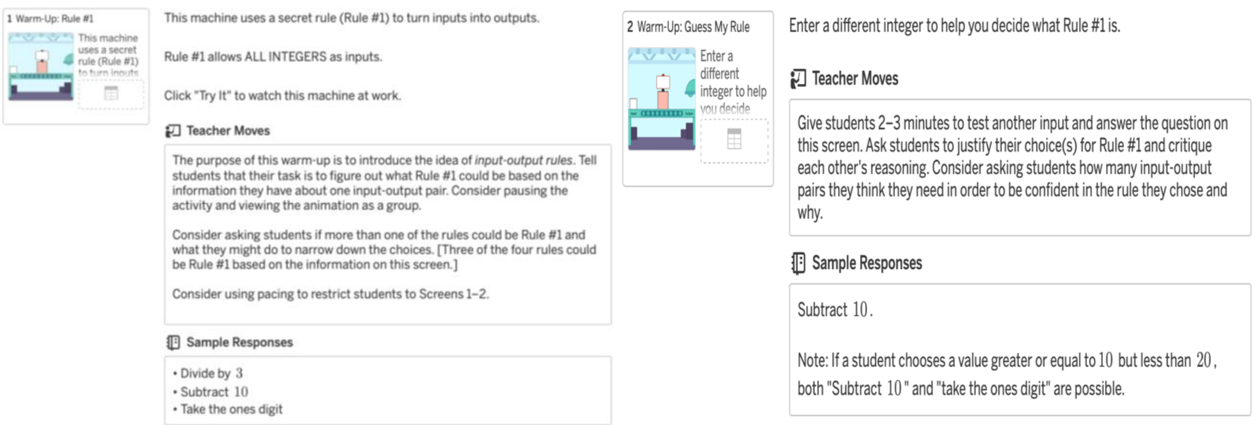


Source: https://www.desmos.com/

Figure 5, shows the activity addressing students understand about the rule that a function has one and only one output for each allowable input. In the first stage, the students are introduced to input-output rules through the animation. Students are asked to guess the rule from the input, which is processed into output (Fig. 6). In this stage, the students learn how to make connections using input-output rules.

Figure 6

Desmos Activities for Function Guess my Rule#1



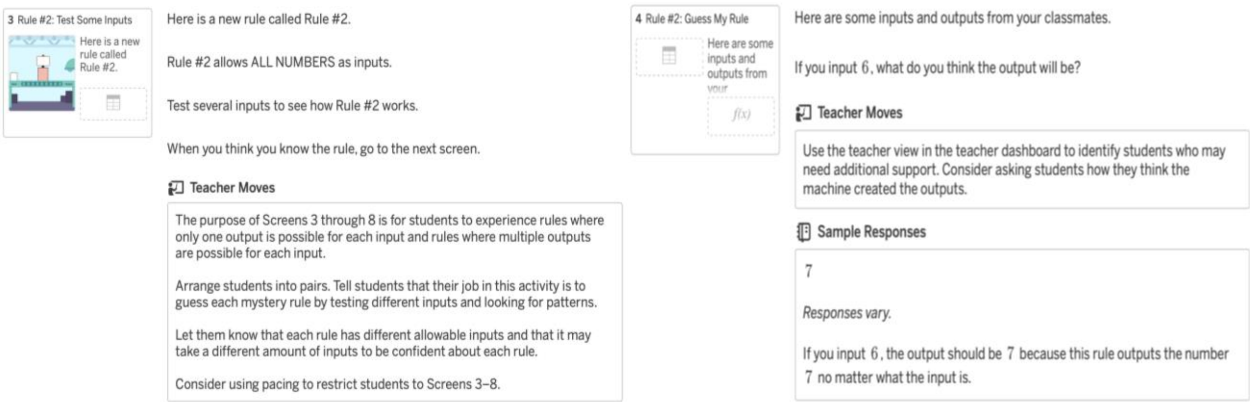
Source: https://www.desmos.com/

The first and second stage are belonged to the rule#1 where the students were asked to guess my rule. The second stage is the confirmation of students' answers in the first stage, where require the students enter the number in the input by themselves. In the second stage, students experiment with numbers to verify their answers. As shown from our previous work in implement this activity (Dewi & Peni, 2023), the students demonstrate two indicators of critical thinking skills: Focus and Reason. Dewi and Peni (2023) stated that the visualisation is crucial for success, as it triggers critical thinking in students. Addition to this, Goldin (1998) stressed the importance of presenting representations in an optimal order that allowed students to build up their ideas gradually. Students will find the image of how the machine works and this skill is linked to enhance their illustrative representation.

The third and eighth stages show three different conditions that might happen in a machine process which are rule#2 (Fig. 7), rule#3(Fig. 8), and rule#4(Fig. 9). In other words, these stages explain examples and non-example of function and lead students to construct their understanding of the meaning of function in the ninth stage.

Figure 7

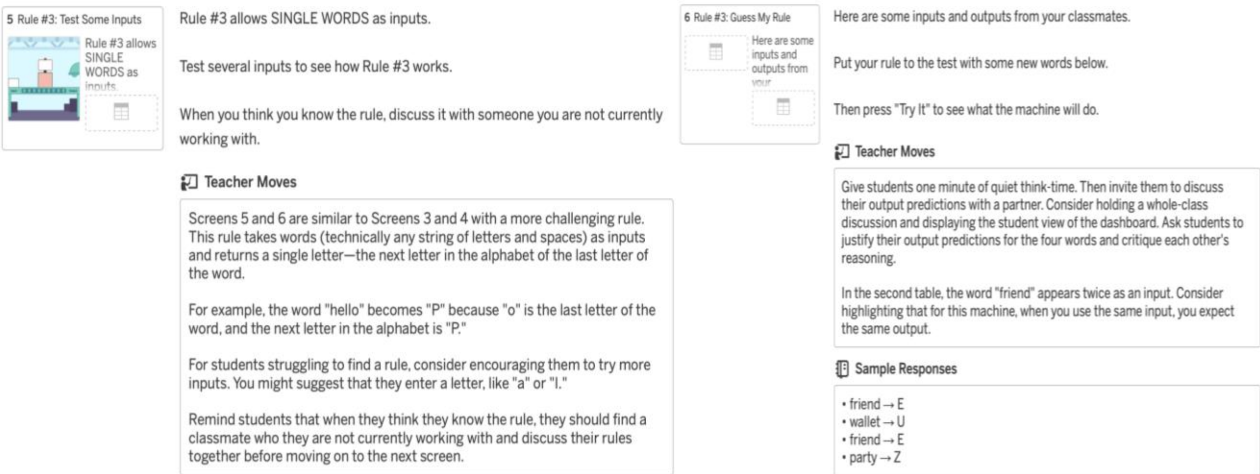
Desmos Activities for Function Guess my Rule#2



Source: https://www.desmos.com/

Figure 8

Desmos Activities for Function Guess my Rule#3



Source: https://www.desmos.com/

Figure 9

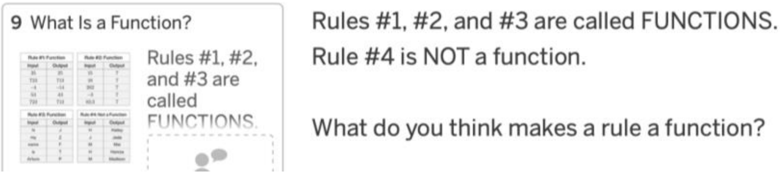
Desmos Activities for Function Guess my Rule#4

Source: https://www.desmos.com/

At the stage 9 (Fig. 10), students have just been introduced to the term function. Activity in this stage, engage students in the process achieving the critical thinking process, distinguishing facts from assumptions, re-examining arguments, and forming conclusions to answer questions (Dewi & Peni, 2023).

Figure 10

Desmos Activities for Function Stage 9

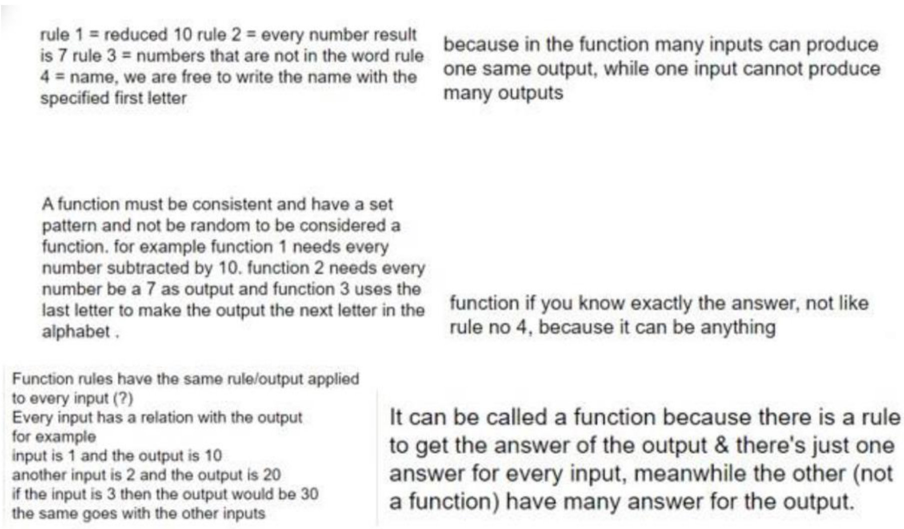


Source: https://www.desmos.com/

Based on the students respond, the summary of responses reveals that there are students who are able to articulate their ideas clearly, there are students who are unable to do so, and there are also students who continue to appear perplexed as to why Rule 4 is stated as not a function.

Figure 11

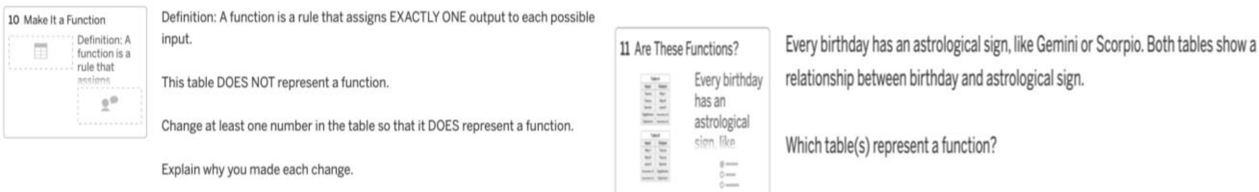
Srudents' Respond to Stage 9



Stage nine exercises enhance students' understanding of function by rephrasing the question "What is function?" to "What do you think makes a rule a function?", encouraging critical thinking and decision-making in the learning process. The tenth stage and eleventh stages are the confirmation stage for students' understanding of basic function concepts. The students were given the real situation related to function and being asked whether it can represent the function concept.

Figure 12

Desmos Activities for Function Stage 10 and 11



Source: https://www.desmos.com/

Evaluation

Table 1 shows the proposed framework based on the learning objectives, well-designed exercises, efficient support, and chances for students to participate in collaborative learning within a mathematics classroom using Desmos. The research journey advances to the evaluation stage, where we rigorously assess the effectiveness of the proposed framework designed to enrich mathematics classrooms through the integration of Desmos. This comprehensive framework, rooted in learning objectives, well-designed exercises, scaffolding, and collaborative opportunities, represents a holistic approach to fostering mathematical understanding and engagement. The evaluation phase involves scrutinising various facets of the framework to determine its impact on students' learning experiences.

Table 1

Proposed Design Framework for Function Classroom

Representation Model	Class Design using Desmos	Indicator for Function
Realistic representational mode	Virtual experiment (guess the rule)	Understanding the concept of function

Manipulative representational mode	Virtual experiment (try to submit input to get an output - confirm the rule).	- Understanding the concept of function - Graphic function
Illustrative representational mode	Virtual experiment of a machine and how it works.	Manipulating and solving function
Linguistic representational mode	The keyword such as input, output, and rule in the virtual experiment.	Translating between representations
Symbolic representational mode	Comparison the real problem (try to compare and determine which problem represent function)	Applying function to real-world problems

Source: author 's own development.

The alignment between a realistic representational mode, class design utilizing Desmos for virtual experiments, and the understanding of the concept of functions is profoundly significant. Realistic representational modes, such as graphical representations in Desmos, provide students with tangible and relatable visuals that bridge the gap between abstract mathematical concepts and real-world applications. When integrated into class design, Desmos allows students to engage in virtual experiments, providing them with opportunities to explore, manipulate, and observe function behaviours in dynamic, interactive settings. This hands-on approach fosters a deeper understanding of functions as students actively experiment and make connections between mathematical abstractions and real-world phenomena. Consequently, this synergy empowers students to grasp complex function concepts more intuitively, ultimately enhancing their mathematical comprehension and problem-solving skills. The convergence of manipulative representational modes, Desmos-driven class design for virtual experiments, and a deep understanding of function concepts, complemented by graphical visualization, creates a comprehensive learning ecosystem. This synergy fosters a multi-dimensional approach to function comprehension, empowering students to navigate the abstract world of mathematics through hands-on exploration, dynamic visualization, and real-world connections.

The interplay between an illustrative representational mode, Desmos-based class design with virtual machine experiments, and manipulating/solving functions represents a cohesive approach. It enables students to explore functions in the context of real-world machinery, enhancing their problem-solving skills by applying mathematical concepts to practical scenarios, thereby strengthening their understanding of functions in a tangible and engaging manner. The convergence of linguistic representational mode, Desmos-centred class design focused on rule exploration within virtual experiments, and the skill of translating between representations creates a comprehensive learning framework. This approach not only deepens students' understanding of mathematical rules but also enhances their ability to articulate and apply these rules across various mathematical contexts, promoting mathematical fluency and problem-solving skills. In the last row of the table 1 show the alignment of symbolic representational mode, Desmos-driven class design for real problem comparisons to discern functions, and the application of functions to real-world scenarios creates a cohesive pedagogical approach. This fosters a deep understanding of functions as students actively analyse and apply them to practical challenges, enhancing their problem-solving abilities and reinforcing the real-world relevance of mathematical concepts.

Discussion

The planning stage is the first stage of the design-based research proposed by Reeves (2006) and modified by the authors and refers to the Goldin (1984) framework in order to get the table 1 as the proposed framework. Technology has been touted as a powerful tool for enhancing mathematics education (Ruiz-Ramos & Kinkead-Clark, 2022; Abu & Kribushi, 2022; Kanandjebo & Lampen, 2022). It has the potential to transform the traditional mathematics classroom by providing students with dynamic and interactive environments for exploring mathematical concepts and visualising graphs of functions. Technology can help students to connect mathematics to real-world problems (Abidin, et.al., 2020), support the development of mathematical thinking and reasoning (Francisco, 2013), and provide opportunities for collaborative learning (Goos et al., 2003; Shechtman et al., 2010). The use of technology in mathematics classrooms can also support differentiated instruction, where teachers can tailor learning experiences in order to meet the needs of individual students (Reeves, 2006). In this case, Desmos can provide students with immediate feedback, adapt to their level of understanding, and create personalized learning paths. Furthermore, Desmos can help to promote equity in mathematics education by providing access to tools and resources that may be otherwise unavailable to students in disadvantaged communities.

After the planning for the class, the tasks should be designed in order to be accessible to students with different levels of mathematical understanding and should provide appropriate scaffolding and feedback. The learning trajectory of learning functions starts from the introduction of concepts. In this case, introducing the function can't be directly done by explaining that "function explanation is something". The activity of the 'Guess My Rule' contains iconic representation (Goldin, 1984), which can bridge children's thinking processes from concrete to abstract things. In using the Guess My Rule activity, the teacher still has a role as a facilitator who can bridge and strengthen each stage of the activity (Leatham & Peterson, 2010; Bell, et al., 2010).

In the implementation stage of the research, an instructional roadmap comprising 11 stages was meticulously devised. These stages were thoughtfully designed to guide students through a progressive and scaffolded learning experience aimed at enhancing their understanding of the concept of functions. Central to this approach is the strategic use of representational systems, which play a pivotal role in shaping students' conceptual development. When employing representational systems, it is recommended that students start with realistic representations, such as illustrated and linguistic ones, and then go on to symbolic representations at the end of their classes (Moskofoglou Chionidou & Vamvouli, 2019) as shown in Figure 6.

The recommendation to commence with realistic representations, such as illustrated and linguistic modes, and eventually transition to symbolic representations, signifies a pedagogical choice that aligns with well-established principles of cognitive development and learning theory. Research in mathematics education underscores the importance of providing learners with concrete and tangible entry points into abstract concepts (Vygotsky, 1978). Realistic representations offer this bridge, enabling students to relate to and visualise mathematical ideas in a way that mirrors their everyday experiences. Beginning with realistic representations allows students to build strong conceptual foundations. These representations provide a concrete context for exploring the behaviour of functions, fostering intuitive connections between mathematical concepts and real-world scenarios. As students' progress through the stages, they gradually internalise and generalise their understanding, moving from the tangible to the abstract. The staged approach is underpinned by the principle of scaffolding, where instructional support is provided and gradually reduced as students gain proficiency (Wood et al., 1976). Starting with realistic representations offers a scaffolded entry point into function concepts, providing learners with the necessary support and guidance to navigate the complexities of functions.

By deferring the introduction of symbolic representations to later stages, one aimed to optimise student engagement and retention. Research indicates that students may struggle to grasp abstract symbols without a solid conceptual foundation (Nesher & Teubal, 1975). By first engaging with realistic representations, students are more likely to approach symbolic representations with greater confidence and comprehension. In conclusion, the implementation approach, characterised by a sequential progression from realistic to symbolic representations, is founded on research-informed principles of cognitive development and pedagogy. This carefully scaffolded journey empowers students to embark on a guided exploration of function concepts, ensuring that they not only grasp abstract mathematical ideas but also develop a profound and enduring understanding of functions that extends to real-world applications.

The evaluation stage where the framework developed, three aspects need to be consider based on the result are representation model, class design using Desmos and the indicator of the Function topic. The proposed framework is able to overcome difficulty in functions for the students. Some common difficulties include understanding the concept of a function, where many students needed help understanding the abstract concept of a function (Tall & Bakar, 1992; Elia et al., 2007), which is the relationship between two variables where each input has only one output that the activity could overcome by Guess my rule in Desmos. Graphing functions also could be taught to the students through virtual experiments, and this activity helps the students accurately understand the relationship between the graph and the function. Manipulating and solving function problems could be overcome when the students practice their representation model in manipulative and illustrative representational modes within the classroom activity. In line with the work of Chandrasegaran et al. (2008) and Bock et al. (2015) that multiple types of representation help students deepen their understanding. Using the linguistic representational mode, the activity designed to determine the keyword helps the students translate between algebraic, graphical, and verbal representations of functions. Leveraging the linguistic representational mode as an instructional strategy is instrumental in facilitating the translation between various mathematical representations of functions (Bossé et al., 2014). By employing activities that focus on determining keywords within mathematical contexts, students are encouraged to discern essential information and establish connections between algebraic, graphical, and verbal representations. This process enhances their ability to bridge the gap between symbolic notation and real-world meaning, aligning with principles of mathematical literacy and relevance which related to the last one is when the students can have a representation model of symbolism, and then they can apply the concept of functions to real-world problems. As students progressively acquire a representation model that encompasses symbolism, they are better equipped to apply the concept of functions to practical, real-world problem-solving scenarios. This holistic approach fosters a deeper understanding of functions while developing the essential skill of transferring mathematical knowledge into meaningful, real-life applications, ultimately empowering students as mathematically proficient problem solvers.

Desmos is a powerful tool for teaching functions in mathematics, but it has its limitations. First, while Desmos excels at graphing functions, it may not fully address the conceptual difficulties that some students encounter when learning functions. Understanding abstract concepts like domain, range, and the nature of functions may still require traditional instruction and clarification. Second, Desmos primarily focuses on graphical representations of functions. It may not adequately support students in mastering the algebraic aspects of functions, such as solving equations, factoring, or working with complex expressions. This limitation means that educators must complement Desmos with other instructional methods to ensure a comprehensive understanding of functions. Third, the effectiveness of Desmos in teaching functions depends on access to technology and a reliable internet connection. Not all students and schools have equal access to such resources, potentially creating disparities in learning opportunities. Lastly, successful integration of Desmos into the curriculum requires proper training and ongoing support for teachers. Without adequate preparation, educators may not fully harness the tool's potential, leading to underutilisation or ineffective implementation. In conclusion, while Desmos is a valuable resource for teaching functions, it should be used alongside other instructional methods to address its limitations and ensure a well-rounded learning experience for students.

Conclusions and Implications

In conclusion, developing a research framework for designing function classes using Desmos is an important step towards enhancing the quality of mathematics education. The integration of Desmos into mathematics classrooms can provide students with dynamic and interactive environments for exploring mathematical concepts and visualising graphs of functions. The research framework for designing functions classes using Desmos will provide mathematics educators with a practical, flexible, and adaptable guide for integrating Desmos into their teaching practices. The resulting framework will enhance student's learning experiences and support the development of their mathematical understanding. As this framework takes root, it promises to not only enrich students' learning encounters but also cater to the diverse needs of educators, ensuring flexibility and adaptability in its application. The outcome of this undertaking is poised to transcend the traditional boundaries of mathematical education, fostering an environment that nurtures heightened comprehension and intellectual growth. Therefore, embracing this research framework signifies a commitment to fostering innovative learning experiences, promoting mathematical prowess, and sculpting a brighter educational landscape for both students and instructors alike.

The findings of this study demonstrated the versatility and applicability of the developed framework for teaching functions with Desmos in various educational settings. Through the systematic design and implementation of the research framework, it was established that its principles and strategies can be effectively adapted to diverse contexts, including different grade levels, educational systems, and cultural backgrounds. This adaptability signified the framework's potential to have a profound and far-reaching impact on mathematics education. By providing educators with a flexible roadmap that incorporates technology-enhanced learning tools like Desmos, one empowered them to address the common challenges in teaching functions across a wide spectrum of environments. As a result, this research not only contributed to the improvement of learning outcomes for students but also promised to enhance the quality of mathematics education on a broader scale. The transferability of the framework underscored its significance as a model for reshaping mathematics instruction, making it more engaging, accessible, and effective in various educational settings worldwide. As educators embrace these principles and strategies, the ripple effect on mathematics education will extend far beyond the confines of the proposed study, ultimately benefiting a diverse and global community of learners.

Suggestions for Future Research

The main prospects for further research on the top are related to the comparative analysis of teaching approaches: to conduct a comparative study that will evaluate the effectiveness of the research framework in designing functions classes using Desmos against traditional teaching methods. This could involve measuring student engagement, comprehension, and overall learning outcomes. Besides, student attitudes and motivation could also be examined in terms of the implementation of Desmos in function classes influences students' attitudes towards mathematics and their motivation to engage with mathematical concepts. This could involve surveys, interviews, and observation of classroom dynamics.

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