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The Validity of Critical-Algebraic Model: A Case Study in Promoting Students' Critical Thinking in Algebra Learning in Junior Secondary Mathematics

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Abstract: This study aims to determine the validity of applying the critical-algebraic model in promoting students' critical thinking skills in algebra learning for junior secondary mathematics education. This study used mixed-method research. The instruments used in this study were surveys, tests, and interviews. A total of 132 Year 7 students were interviewed; the quantitative data was collected through the survey, which provided information about how students defined themselves using critical skills in learning algebra. Nine mathematics teachers were interviewed to collect information about critical thinking skills, algebraic learning, and the challenges of learning algebra based on the quantities of data needed to design and write the pre-test and post-test. Assessment phases, including pre-test and post-test, were used to assess students' levels of critical thinking skills before and after the implementation of the model. Two classes - one class (28 students) served as a control group, and another (28 students) served as an experimental group; both classes attended the pre-tests, and the

experimental group was given a post-test after implementing the model. Mean and standard deviation were analysed via SPSS to prove the model's validity. This study used the Holistic Critical Thinking Scoring Rubric to assess critical thinking levels. The resulting data indicate that 36% of the students understand critical thinking skills in algebra. Through the application of the model, 25.76% of students improved their critical thinking skills, and the number of students whose critical thinking skills level is under satisfactory has decreased from 22% to 8.70%, which can be concluded that appropriate implementation of the model will provide effective teaching and learning. These findings have implications for developing students' critical thinking skills via applying the model. It also states that teachers are essential in promoting students' critical thinking in algebra learning.

Keywords: Algebra Teaching, Algebraic Learning, Critical-Algebraic Model, Critical Thinking Level.

Introduction

Critical thinking is a process of analysing and assessing thinking. The concept of critical thinking can be expressed in a variety of definitions, depending on one's purpose. According to Liu (2023a), in mathematics education, critical thinking may involve logical reasoning and skills while separating facts from opinions. Individuals who think critically can understand logical connections between ideas, detect mistakes in induction and deduction reasoning, and make rational decisions while solving problems. Critical thinking has become an essential competency ability and survival skill for students to face the 21st century (Liu, 2023a); critical thinking skills should be part of student learning, and schools should be responsible for developing and assessing critical thinking skills through the teaching and learning process (Firdaus et al., 2015).

Developing critical thinking skills in mathematics has been emphasised by research and in the curriculum of mathematics (Fuad et al., 2015; Liu, 2023a; Liu, 2023b; Liu, 2024; NCTM, 2000). However, developing critical thinking skills is complex and complicated due to a lack of agreement on the definition of critical thinking and an associated framework for its development (Ku, 2009; Lee et al., 2024). Teaching methods should be one factor that affects how critical thinking is developed in the classroom. Subjects like mathematics are more likely taught with teacher-centred learning, and students can be regarded as passive learners who listen and copy what has been written and said by the teacher. Students might not learn efficiently because they have different learning styles. The student-centred method allows students to explore the question through discussion and communication; students need to use various strategies to complete the task; it will enable students to create knowledge instead of passively receiving information and encourages deep learning. "Without taking away from the important role played by the teacher, it is helpful to remember that what the student does is more important in determining what is learned than what the teacher does" (Shuell, 1986, p. 492). While this teaching method might seem time-consuming, students gain a deep understanding of the question, as they are doing rather than just copying down what they are told to do (Belecina & Ocampo, 2018). However, the heavy workload for teachers to cover the curriculum makes it a challenge to apply a student-centred approach frequently. Often, problems can have multiple solutions, but students will only try to find solutions using methods they are familiar with. This issue needs to be overcome to balance workload and time constraints. Thus, an effective way of developing students' critical thinking skills in mathematics learning is vital for effective teaching and learning.

Research Questions

The purpose of this study was to address the information on the construct of the connection between critical thinking skills and algebra learning. By doing so, this study explored students' critical thinking levels, and what critical thinking skills students should master, further evaluating the validity of the critical-algebraic model to provide a guide for teaching algebra with critical thinking in secondary schools. Research questions that were explored in this study are:

1. How do junior students conceptualise critical thinking in their algebra context?

2. How does the Critical-Algebraic Model promote students' critical thinking skills in algebra learning?

In this context, when solving algebraic questions, this study aims to investigate whether junior secondary students display critical thinking skills through surveys. The results of this study are important for assessing how the students think when solving algebraic questions and observing how they demonstrate critical thinking skills. Additionally, the study will help teachers to determine if the student's performance is reflected in their critical thinking skills; it also provides information for the teachers' self-assessment in their pedagogical practice. Additionally, it will improve the quality of algebra teaching in junior secondary schools.

Literature Review

Based on the problem statement, to know how to teach algebra, the teaching approach and challenges that teachers face need to be reviewed, and how to develop critical thinking skills in algebra learning needs to be explored.

Algebra Teaching

Algebra is one of the important branches of mathematics, especially for secondary mathematics from Year 7 to Year 12. Algebra plays a key role in mathematics acquisition, which explains the importance of teaching students basic algebra concepts (Theodora & Hidayat, 2018). Cai et al. (2005) stated that algebra was considered an important 'gatekeeper' in mathematics learning because it correlates with indicators of critical thinking skills, which are the essential skills addressed in different areas of the mathematics curriculum.

When it comes to teaching and learning algebra, there are two synthesised obstacles faced by both teachers and students. One obstacle is content understanding. Algebra includes many terminologies, such as variables and pronominal, like terms, coefficients, and algebraic expressions. Students need to master the basic conceptions before using algebraic knowledge and methods to solve problems, which involve multi-steps in dealing with the processes. To master these, students must practice after teachers' explanations and demonstrations. Students can better generalise patterns from statements than they could by forming equations containing unknown quantities (Kieran, 2004). Challenging content students face includes but is not limited to "turning the problem into mathematical", "understanding the complex relationships between variables and other values" and "how to figure out the relationships between terms" (Liu, 2024, p. 2453). These are due to the student's lack of understanding of the relationships of certain conditions in given problems and their incomplete understanding of the meaning of variables. The other obstacle is teaching methods. Traditional math courses often prepare students to memorise concepts rather than solve problems and understand the reasoning behind math skills (Rossman, 2006). Algebra teachers, like other subject teachers, tended to follow a textbook (Kieran, 1992), focused on calculating results, using rules, and following procedures (Gordillo & Godino,

2014), rather than guiding students to transform knowledge from concrete to abstract, from elementary calculations to algebraic equations, and from experience to concepts. For example, McNeil and Alibali (2005) found that students often do not tend to see a relational understanding of the equal sign unless they have contextual support. Therefore, teachers need to examine the contexts, explain the contexts, and clarify the problems to support students' understanding of the algebraic contents.

National Council of Teachers of Mathematics (NCTM, 2000, p. 1) articulates the four major themes run through algebra learning in Principles and Standards that students should be able to (1) understand patterns, relations, and functions; (2) represent and analyse mathematical situations and structures using algebraic symbols; (3) use mathematical models to represent and understand quantitative relationships; (4) analyse change in various contexts. The Principles and Standards expect mathematically proficient students to master all these skills in algebra, to do that, teachers need to apply an effective method to teach algebra, not follow the traditional way, but combine different pedagogical approaches with different algebraic contents, to facilitate students to think critically, for example, in explaining correspondences between equations, relationships between variables, exploring using diagrams or pictures to help conceptualise and solve problems. To evaluate their results, students are trained to check themselves by asking, “Does this make sense?” “Do the values on both sides of the equation equal?” Teachers play an important role in cultivating students' thinking skills in dealing with algebraic questions, guiding students in mastering the approaches to solve complex problems and identifying the correspondents between different approaches. Teaching styles such as student-centred, discussion-based activities that utilise constructivist ideas, authentic situations, critical thinking skills, and engagement with real-world problems can emphasise knowledge and approaches to learning algebra. Therefore, effective teaching is a prerequisite to obtaining effective learning.

Develop Critical Thinking in Algebra

Aspects of traditional mathematics content, including functions and algebra, including algebraic expressions, equations and inequalities, tables, and graphical representations are central to describing, modelling, and explaining phenomena (Vale & Barbosa, 2023). To be fluent in these aspects, students must focus on the questions that are specific to these contents. Most schools do not teach students to think critically or problem-solving skills. Worksheets only contain facts that require students to remember, so the assessment ability test is limited to memorisation (Yennita et al., 2018). This problem is caused by students being confused about the usage of concepts. Therefore, students try to get the final answer instead of understanding how to solve it. To improve this situation, teachers need to take on the responsibility of teaching higher-order skills at pre-tertiary levels, which allows the growth of critical thinking skills levels to occur (Awofala & Lawal, 2022), this enables students to actively think, reflect and evaluate their results rather than on passive acceptance. There are few reports to deal specifically with algebra teaching. Critical thinking skills appeared to be a key factor in promoting students’ independent thinking, thus developing students’ mathematical thinking (Foundation for Critical Thinking, n.d.; Jacob, 2012; Liu, 2023a; Liu, 2023b). Evaluation of critical thinking skills in mathematics must pay attention to students’ thinking processes while solving problems in mathematics, not just the result of the answer. Components of critical thinking skills in algebra can be measured by five aspects (Liu, 2024). Therefore, to develop critical thinking skills in algebra learning, it is also necessary to make assessment instruments for the components of critical thinking associated with the algebra context. In general, when developing critical thinking skills in algebra, teachers need to be aware that the process of learning mathematics is more important than the results, mathematical thinking is more important than following the teacher's procedures, and critical thinking is more important than accepting what the teacher tells you to do.

There are different methods or models for teaching algebra efficiently. For example, Augmented Reality (AR) for promoting algebra learning (Rahardjoni et al., 2020) and using a video game app to explore the impact of students' algebraic learning (Siew et al., 2016). However, there were different limitations to using these models in algebra (Liu, 2023b, 2024). To master algebra, the ability of each component of students' algebraic thinking requires the development of critical thinking skills; Liu (2024) designed a critical-algebraic model to provide an innovative pedagogical approach that aims to develop students' critical thinking through algebra learning. It is worthwhile to investigate the validity of the model and further guide its implementation in mathematics education.

Method

Research Design

This study employed the experimental method. It used several data-gathering instruments in the study: a survey and the pre-test of students' critical thinking skills, and a post-test was applied to collect data for comparing the progress of students' critical thinking levels. The mixed method was used in this study. The research process combined quantitative and qualitative research (Henline-Hall, 2024). Data collection was done through surveys, tests, and interviews. Quantitative methods were used to collect data on the students' understanding of critical thinking skills in algebraic questions. Interviews with middle school mathematics teachers were conducted to collect data on student's critical thinking skills when solving algebraic problems. In contrast, the qualitative method was used to obtain information on evaluating the model's effectiveness. This study used the critical-algebraic model (Liu, 2024) to assess and develop critical thinking in algebra; the Holistic Critical Thinking Scoring Rubric was used to analyse students' critical-thinking levels on the pre-and post-test.

Research Instruments

The first phase was the research design (Shown in Table 1). When designing the survey, the items were constructed based on the mathematics algebraic content of the Year 7 level. Other findings were considered, such as students' deficiencies in generalising patterns and using symbols (Fakhrunisa & Hashanah, 2020; Patton & Santos, 2012). Questions such as general expressions resulting from arithmetic or geometric patterns, numerical sequences or numerical relationships and equations that include unknowns to represent problem situations (Kieran, 2004) were included in the pre-test.

The test consisted of 13 items designed and modified from textbooks and reference books. To assess whether students understood the concepts, they were asked to write number patterns; based on the information of word questions, students were asked to provide evidence of reasoning and logic when answering the questions. For some questions, students were required to provide all the possible results through inductive and deductive reasoning and questioning. Students need to demonstrate an understanding of mathematical concepts, recognise the connections between values and the ability to solve mathematical problems. The critical thinking pathway was assessed during the process, and students' critical thinking skills were analysed using the pre-test results.

Table 1

Research Design

Method	Aim
Pre-test	Collect data on student's critical thinking skills/levels in algebra. Analysis of students' critical thinking skills level.

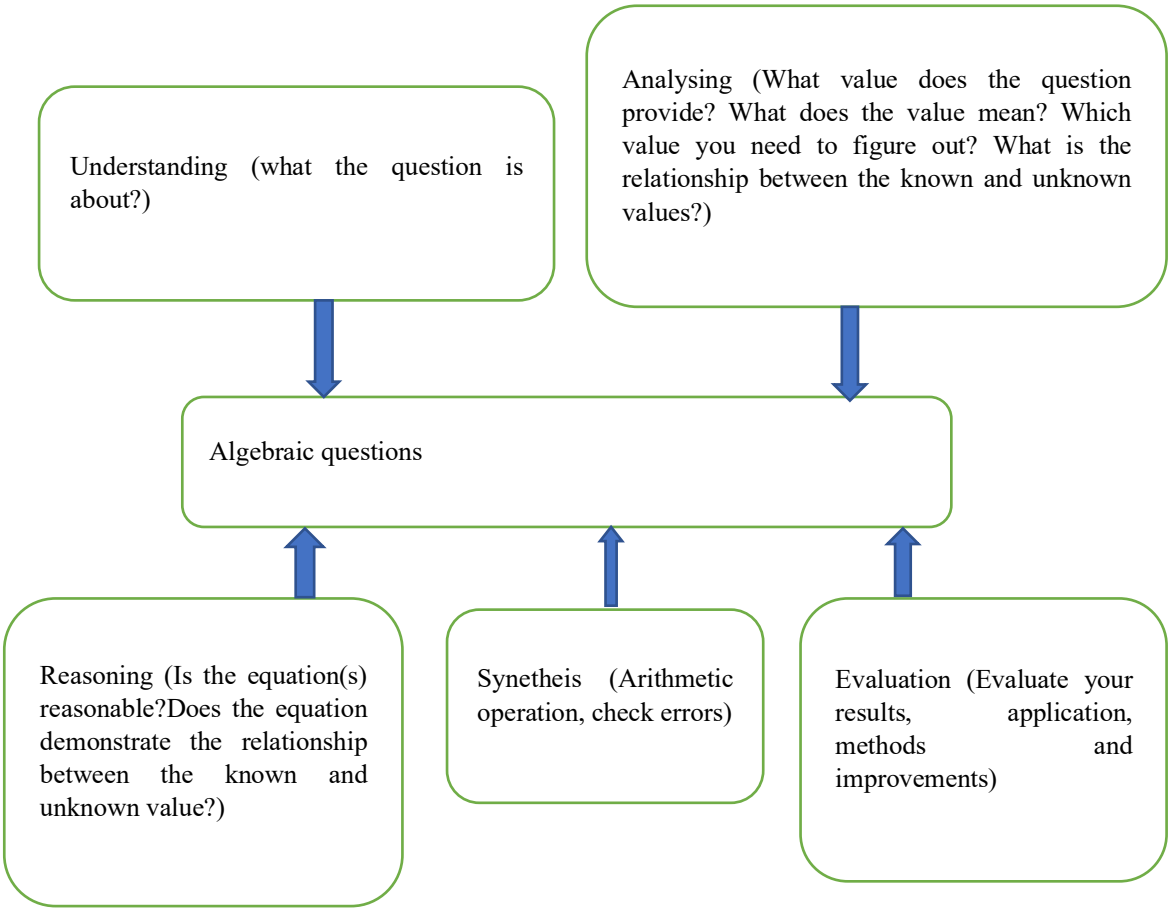
Implement critical-algebraic model	Explore an effective method for supporting students in developing critical thinking skills in algebra.
Discussion with participants Classroom observation	Apply the model in groups in different classes.
Post-test	Collect data on student's critical thinking skills level.
Analysis of the results (comparing students' level, and improvement)	To analyse the relationship between critical thinking skills and mathematics achievement.

Source: Authors' development.

The second phase was the model application (Shown in Figure 1). This study applied the critical algebraic model (Liu, 2024) in daily practice. Participants had a pre-test before implementing the model and a post-test after using the model to evaluate the effectiveness of the pedagogical practice. Before implementing the model, teachers' opinions on the connection between algebra and critical thinking skills and students' understanding of critical thinking were collected, and pedagogical knowledge and methods were discussed as preliminary work before conducting the study. The experimental design involved two classes: the experimental and control classes. The control class was conducted in the pre-test without applying the model, while the experimental class conducted the model. During the learning process of algebra, the teacher of the experimental class would guide students according to the structure of the model to promote their understanding and learning.

Figure 1

Critical-Algebraic Model



Source: Liu (2024, p. 2456).

The third phase was the assessment phase. All students who participated in the pre-test in both classes were analysed for their critical thinking skills. Only the experimental class was analysed to observe improvements. The effectiveness of the learning model was determined by comparing the critical thinking skill level and algebraic post-test results. Additionally, the critical algebraic model was further validated by mathematics teachers. After implementing the model, a post-test will be applied during this phase to collect data on student results. The model's effectiveness was based on two aspects 1) increased percentage of levels and 2) the number of students showing improvement.

The validation results of the algebraic academic results and a critical thinking assessment were analysed, and teachers and students were involved in providing feedback about the model. The mathematics teacher who applied the model was interviewed to get responsive reflection about the validation of the model. For students, the first step was for the whole class (28 students) to participate in the trial of the model; the next step was a limited trial phase. At this stage, eight students who had achieved outstanding improvement in using critical thinking skills while solving algebraic questions were tested. Then, students chosen for the limited trial phase answered a questionnaire of responses to assess the practicality of the test material. Meanwhile, the teacher who applied the model was given an observation sheet to determine the learning devices used. The post-test data will be compared to the pre-test data to explore the relationship between students' critical thinking levels and algebraic understanding, and this will also provide information about the model's validity.

Participants

This study took place at a Christian college located in Victoria, Australia. A total of 132 Year 7 students participated in the survey about their perceptions of critical thinking in learning algebra. 56 students participated in the pre-test, 28 in the control group and the rest in the experimental group to test the model's effectiveness. 9 Mathematics teachers participated in the semi-interview to collect details of the challenge of algebra learning.

Data Collection and Analysis

In the study, frequency and percentage, weighted mean, standard deviation, and data were analysed using the statistical software SPSS when analysing the data's mean, median, and distribution.

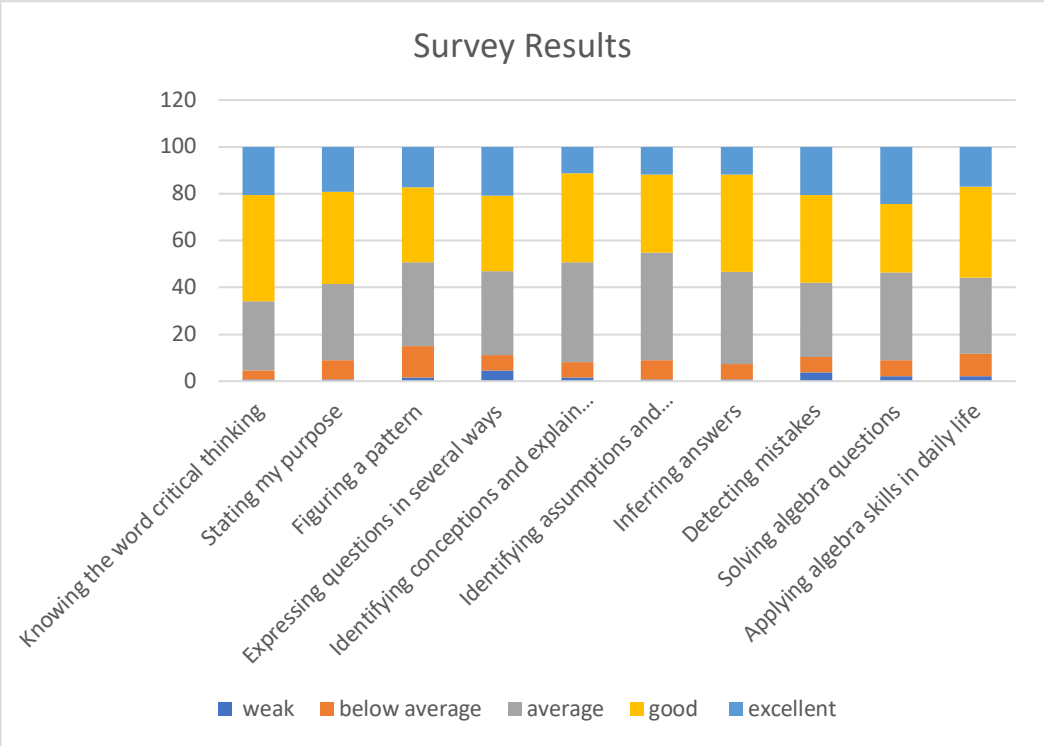
Results

Question 1: How do junior students conceptualise critical thinking in their algebra context?

There were 132 Year 7 students who did the survey online, and questions involved their perceptions of critical thinking skills in learning algebra to target skills that students are expected to be aware of during the process of algebra learning.

Figure 2

Students' Perceptions of Critical Thinking Skills in Algebra Learning



Data showed that critical thinking skills are not a strange terminology for students; 36% of students thought that their understanding of critical thinking skills in algebra is average, as in Australian and Victorian mathematics curricula, critical thinking is stated, and teachers might mention it during class time. 38% of students chose good when evaluating their skills in the survey. The weak points are "expressing questions in several ways" (4.48%) and "detecting mistakes" (3.68%). The other responses are below average, such as "figuring a pattern was occupied" at 13.43%. When solving a mathematics problem, "stating my purpose" was only 8.15%, " identifying assumptions and determining whether they are justifiable" was the same at 8.15%, "applying algebraic skills to solve problems in my daily life" was 9.56%, which is below among all the skills.

Question 2: How does the Critical-Algebraic Model promote students' critical thinking skills in algebra learning?

Table 2

The Range of Scores of Pre-test in Critical Thinking Skills

Class A Range of score %	Level of students' critical thinking skills	Number of students	Frequency
80-100	Excellent	2	7.41%
60-79	Good	13	48.15%
40-59	Fair	4	14.81%
20-39	Poor	2	7.41%
1-19	Very poor	6	22.22%
Total		27	100%

Class B Range of score %	Level of students' critical thinking skills	Number of students	Frequency
80-100	Excellent	1	5.26%
60-79	Good	2	10.53%
40-59	Fair	8	42.11%
20-39	Poor	8	42.11%
1-19	Very poor	0	0 %
Total		19	100%

Table 3

The Range of Scores of Post-Test in Critical Thinking Skills

Experimental group Class A Range of score %	Level of students' critical thinking skills	Number of students	Frequency
80-100	Excellent	5	21.74%
60-79	Good	11	47.83%
40-59	Fair	4	17.39%
20-39	Poor	1	4.35%
1-19	Very poor	2	8.70%
Total		23	100%
Control group Class B Range of score %	**Level of students' critical thinking skills**	**Number of students**	**Frequency**
80-100	Excellent	2	10.53%
60-79	Good	2	10.53%
40-59	Fair	7	36.84%
20-39	Poor	7	36.84%
1-19	Very poor	1	5.27%
Total		19	100%

Discussion

The results of the survey showed that the students were found kind of difficulties in understanding critical thinking in algebra. This occurred when answering questions about specific algebra questions; students did not show confidence in their answers. The reason for this is the misconception, which leads to a lack of understanding in further algebra lessons; this finding is similar to the other research's findings (Carpenter et al., 2003; Widodo et al., 2020). As mathematics knowledge is constructive, a limited conception of what algebra is will make students struggle with algebra questions; they cannot build on their knowledge and cannot identify the relational concepts and relationships, thus, they need more emphasis on the understanding of the concepts and contents, and how to learn it effectively and critically, students need much more support to prepare them to reach a higher proficiency in mathematics (Teoh, et al., 2024), this is similar as stated by Turan et al. (2019). When doing algebraic questions, directing the ideas given by a student to be well received by others, and justifying the conclusions provided by students properly (Rohati et al., 2022), teachers' encouragement and cultivating a critical and inquiring class environment is vital for students to develop their ideas when completing tasks. The survey finding matches the prior studies, as Liu (2024) states that teachers have a responsibility to promote and support students to develop critical thinking skills in mathematics

learning; however, when teaching algebra, teachers rely on textbooks (Kieran, 1992) therefore effective pedagogical practice is needed to facilitate the development of critical thinking skills.

The research findings showed that there was a significant difference between the experimental group and the control group. The pre-test and post-test aimed to measure students' critical thinking skills in algebraic problem-solving. The evaluation of critical thinking skills in mathematics should focus on the processes used to solve problems, not just the results of the answers. Therefore, in both tests, the questions evaluate not only students' mathematical knowledge in algebra but also assess the level of critical thinking. For example, students are provided with a situation related to the pattern, and then students are asked to identify the rules in the pattern; further, they are required to generalise the given pattern. The regulations for generalising patterns are important because they provide insights into how students first visualise different terms, and their process of how to generalise relationships between terms depends on their ability to use reasoning and logic skills (Wilkie & Clarke, 2015). The Holistic Critical Thinking Scoring Rubric (HCTSR) was applied to analyse students' critical levels in the pre-test and post-test. Data showed that students face challenges with questions when applying algebraic expressions to perform complex procedures (Siew et al., 2018, cited by Rahardjoni et al., 2020). Students need to understand the description and have the ability to change the words' description to algebraic equations. After they figure out the results, they need reasoning skills to test their answers. If students do not master critical thinking skills, they can't show their work out properly. After implementing the model (data were shown in table 2 and 3), the percentage of students from the controlled class, who achieved excellent critical thinking levels from 7.4% increased to 21.74%, which demonstrated that this model works effectively in supporting students to develop critical thinking skills in algebra learning, this can be seen from the results of the test- the last question of both the pre-test and post-test needs students to analyse the questions (shown in figure 2), students can apply the skills that the model provided to their practice. The percentage of good critical thinking levels did not change. The significant finding is that the percentage of students who are at an inferior level in the pre-test 22.22% decreased to 8.70% in the post-test, which confidences teachers in the future planning and teaching. Overall, students who obtained a good or excellent level of critical thinking increased from 55.56% to 69.57%. Data found that some students gave up and did not even try the last question, which teachers should know that critical thinking should be embedded into daily teaching and practising. There should be virtually no distinction between how critical thinking is applied in tests and how it is applied in embedded daily activities. Data from the interview and teacher's feedback on the model point out that providing an appropriate learning model can train students to get used to critical thinking and also train students to bring up the potential of critical thinking and learning skills. The critical algebraic model provides students with opportunities to improve their abilities in learning activities and develop their algebraic and critical thinking skills. The critical-algebraic model combines teacher-centred and student-centred in practice because it will be difficult to develop critical thinking skills when only using teacher-centred learning with algebra (Duron et al., 2006). According to the data, the critical-algebraic model improves critical thinking skills in learning algebra.

Conclusion

According to research results, this study has obtained several findings on students' critical thinking in algebra learning. First, the students' understanding of critical thinking was characterised by their analysis of the problems given in the survey. They were given related algebraic questions to check their application of critical thinking skills when solving problems; however, not every student was aware of the skills that they needed to use. Second, Junior students still need to improve their ability to think critically in mathematics, teachers play an important role in facilitating students in learning and developing critical thinking in algebra; effective learning enables students to learn and acquire

knowledge in the classroom through creative teaching methods, through the effort of the teacher, special assistance for students to develop critical thinking skills of interference, deduction, interpretation and application. Third, based on the data, the critical-algebraic model has positive results for improving critical thinking skills in learning algebra. The percentage of students who obtained good or excellent levels in the post-test was 69.57% compared with 55.56% in the pre-test, the increased percentage was 25.76%. Results revealed that students who learned algebra using the critical-algebraic model showed significantly higher mean scores in critical thinking skills toward algebra compared to the control group.

This study's limitation is that the learning model tested for one-year implementation can not be generalised to determine its validity. If external validity can be tested in the future, the model will be revised as needed to ensure its implementation's effectiveness.

This study suggests that the critical algebraic model can be used to help junior school students learn algebra. The model will support students to develop critical thinking skills in mathematics education. From this research's findings and discussion, future research is recommended: what education practices can be positioned to nurture the augmentation of critical thinking skills in learners? A pedagogy focused on developing deep knowledge and understanding of concepts and skills (NSW DET, 2003) in algebra learning. A more directive approach that considers students' learning preferences and infuses thinking and learning skills throughout this model.

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Conflict of Interest

None.

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References

Awofala, A. O. A., & Lawal, R. F.(2022). The relationship between critical thinking skills and quantitative reasoning among junior secondary school students in Nigeria. *Jurnal Pendidikan Matematika (Kudus), 5*(1), 1–16. https://www.academia.edu/download/89303543/pdf.pdf

Belecina, R. R., & Ocampo, J. M.(2018). Effecting change on students critical thinking in problem solving. *International Journal for Educational Studies, 10*(2), 109–117. https://journals.mindamas.com/index.php/educare/article/view/949

Cai, J., Lew, H., Morris, A., Moyer, J., Ng, S., & Schmittau, J. (2005). The development of students algebraic thinking in earlier grades: A cross-cultural comparative perspective. *Zentralblatt für Didaktik der Mathematik, 37*(1), 5–15. https://doi.org/10.1007/bf02655892

Carpenter, T, P., Franke, M, L.,& Levi, L.(2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Heinemann.

Duron, R., Limbach, B., & Waugh, W.(2006). Critical thinking framework for any discipline. *International Journal of Teaching and Learning in Higher Education, 17*(2), 160–166. https://valenciacollege.edu/faculty/development/courses-resources/documents/ctframeworkarticle.pdf

Fakhrunisa, F., & Hasanah, A. (2020). Students' algebraic thinking: A study of mathematical modelling competencies. *Journal of Physics: Conference Series, 1521*(3), Article 032077. https://doi.org/10.1088/1742-6596/1521/3/032077

Firdaus, F., Kailani, I., Bakar, M. N. B., & Bakry, B. (2015). Developing critical thinking skills of students in mathematics learning. *Journal of Education and Learning (EduLearn), 9*(3), 226–236. https://doi.org/10.11591/edulearn.v9i3.1830

Foundation for Critical Thinking. (n.d.). National council for excellence in critical thinking https://www.criticalthinking.org/pages/the-national-council-for-excellence-in-critical-thinking/406

Fuad, J., Ardana, I. W., & Kuswandi, D. (2016). Increasing critical thinking skill through class debate. In *Proceedings of the 1st UPI International Conference on Sociology Education (UPI ICSE 2015)* (pp. 38–42). Atlantis Press. https://doi.org/10.2991/icse-15.2016.9

Gordillo, W., & Godino, J. (2014). Preservice elementary teacher's thinking about algebraic reasoning. *Mathematics Education, 9*(2), 149–164. https://doi.org/10.29333/iejme/287

Jacob, S. M. (2012). Mathematica achievement and critical thinking skills in asynchronous discussion forums. *Procedia – Social and Behavioral Sciences, 31*, 800–804. https://doi.org/10.1016/j.sbspro.2011.12.144

Henline-Hall, J. (2024). Introduction to quantitative and qualitative research methods. *Radiologic Technology, 96*(1), 45–56. http://www.radiologictechnology.org/content/96/1/45.extract

Kieran, C. (1992). The learning and teaching of school algebra. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 390–419). New York: Macmillan Publishing Company.

Kieran, C. (2004). Algebraic thinking in the early grades: What is it?. *The Mathematics Educator, 8*(1), 139–151. http://surl.li/gdzodz

Ku, K. Y. (2009). Assessing students' critical thinking performance: Urging for measurements using multiresponse format. *Thinking Skills and Creativity, 4*(1), 70–76. https://doi.org/10.1016/j.tsc.2009.02.001

Lee, N. Y., Wang, Z., & Lim, B. (2024). The development of critical thinking: What university students have to say. *Teaching in Higher Education, 29*(1), 286–299. https://doi.org/10.1080/13562517.2021.1973412

Liu, W. (2023a). Exploring Chinese secondary teachers' perception of critical thinking in mathematics teaching. In *The Asian Conference on Education & International Development 2023 Official Conference Proceedings* (pp. 269–280). https://doi.org/10.22492/issn.2189-101X.2023.23

Liu, W. (2023b). Critical thinking skills for Chinese teachers: A study of mathematics teachers' perceptions. *PUPIL: International Journal of Teaching, Education and Learning, 7*(2), 1–16. https://doi.org/10.20319/pijtel.2023.72.0116

Liu, W.(2024). An innovative model to promote secondary students' critical thinking skills in algebra learning. *International Journal of Social Science and Human Research, 7*(4), 2448–2458. https://doi.org/10.47191/ijsshr/v7-i04-28

McNeil, N., & Alibali, M. W. (2005). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition and Development, 6*(2), 285–306. https://doi.org/10.1207/s15327647jcd0602_6

NCTM. (2000). *Principles and standards for mathematics*. National Council of Teachers of Mathematics. https://www.itws.org/NCTM-ContentProcessCoreStandards.pdf

NSW DET. (2003). *Quality teaching in NSW public schools: Discussion paper*. New South Wales: Department of Education and Training. URL: http://www.darcymoore.net/wp-content/uploads/2012/02/qt_EPSColor.pdf

Patton, B., & Santos, E. (2012). Analyzing algebraic thinking using "guess my number" problems. *International Journal of Instruction, 5*(1), 5–22. https://dergipark.org.tr/en/download/article-file/59749

Rahardjoni, A. S., Hasanah, I. N., & Nugraheni, M. S. (2020). Developing critical thinking competence in algebraic thinking using augmented reality for junior high school. *PRISMA, Prosiding Seminar Nasional Matematika, 3*, 497–503. https://journal.unnes.ac.id/sju/prisma/article/download/37768/15509

Rohati, R., Kusumah, Y. S., Kusnandi, K., & Marlina, M. (2022). How teachers encourage students' mathematical reasoning during the Covid-19 pandemic?. *Jurnal Pendidikan Indonesia, 11*(4), 715–726. https://doi.org/10.23887/jpiundiksha.v11i4.52756

Rossman, S. (2006). Overcoming math anxiety. *Mathtitudes, 1*(1), 1–4. https://www.fau.edu/education/centersandprograms/mathitudes/documents/math-anxiety-research-paper-2.pdf

Shuell, T. J. (1986). Cognitive conceptions of learning. *Review of Educational Research, 56*(4), 411–436. https://doi.org/10.3102/00346543056004411

Siew, N. M., Geofrey, Jolly, & Lee, B. N. (2018). Students' algebraic thinking and attitudes towards algebra: the effects of game-based learning using Dragonbox 12+ App. *The Research Journal of Mathematics and Technology, 5*(1), 66–79. https://www.academia.edu/download/46188339/2dragonbox.pdf

Teoh, S. H., Mohamed, S. R., Hong, J. B. Z., Rameli, M. R. M., Alhassora, N. S. A., & Mazlan, A. N. (2024). Navigating mathematical challenges: Enhancing mathematical buoyancy among rural students for tertiary-level preparedness. *Asian Journal of University Education, 20*(1), 173–184. https://doi.org/10.24191/ajue.v20i1.26027

Theodora, F. R. N., & Hidayat, D. (2018). The use of realistic mathematics education in teaching the concept of equality. JOHME: Journal of Holistic Mathematics Education, 1(2), 104–113. https://ojs.uph.edu/index.php/JOHME/article/view/913

Turan, U., Fidan, Y., & Yıldıran, C. (2019). Critical thinking as a qualified decision-making tool. *Journal of History Culture and Art Research, 8*(4), 1–18. https://hdl.handle.net/11467/4069

Vale, I., & Barbosa, A. (2023). Active learning strategies for an effective mathematics teaching and learning. *European Journal of Science and Mathematics Education, 11*(3), 573–588. https://doi.org/10.30935/scimath/13135

Widodo, S. A., Irfan, M., Trisniawati, T., Hidayat, W., Perbowo, K. S., Noto, M. S., & Prahmana, R. C. I. (2020). Process of algebra problem-solving in formal student. *Journal of Physics: Conference Series, 1657*(1), Article 012092. https://doi.org/10.1088/1742-6596/1657/1/012092

Wilkie, K. J., & Clarke, D. M. (2016). Developing students' functional thinking in algebra through different visualisations of a growing pattern's structure. *Mathematics Education Research Journal, 28*, 223–243. https://doi.org/10.1007/s13394-015-0146-y

Yennita, K, I., Gibran, & Irianti, M. (2018). Development of worksheet based on high-order thinking skills to improve students' high-order thinking skills. *Journal of Educational Sciences, 2*(1), 37–45. https://www.academia.edu/download/91546752/4617.pdf