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The Use of GeoGebra as an Instrument to Facilitate Grade 11 Learners' Understanding of Cyclic Quadrilateral and Tangent Theorems: A Case Study

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Abstract. This study explores the use of GeoGebra as a teaching tool to improve Grade 11 learners' understanding of cyclic quadrilaterals and tangent theorems in circle geometry. Persistent challenges in Euclidean geometry in South African schools are often linked to traditional teaching methods that rely heavily on memorisation and offer limited opportunities for meaningful visual engagement. In response, a qualitative case study was conducted with one Grade 11 mathematics teacher and thirty-nine learners at a public secondary school in the Western Cape. The intervention involved three GeoGebra-supported teaching cycles over three weeks. Data were collected through classroom observations, interviews with learners and the teacher, and a post-intervention learner survey. The analysis focused on how learners utilised GeoGebra as a tool for geometric reasoning and understanding. The findings show that GeoGebra enhanced learners' ability to visualise geometric relationships, improved their conceptual understanding, and increased their engagement during circle geometry lessons. Learners demonstrated a greater ability to explore

relationships, form conjectures, and collaborate during problem-solving activities. Evidence further indicated that learners shifted from seeing GeoGebra as a simple drawing tool to using it as a reasoning resource to test and verify geometric ideas. While notable advances in conceptual understanding were observed, additional support was still necessary to strengthen learners' ability to construct formal proofs. The study concludes that GeoGebra can be an effective instructional tool for fostering conceptual understanding in circle geometry when integrated into a learner-centred teaching approach.

Keywords: GeoGebra, Circle geometry, Cyclic quadrilaterals, Tangent theorems, Instrumental genesis, Constructivism.

Introduction

Euclidean geometry plays a critical role in the development of deductive reasoning, spatial visualisation, and mathematical justification in secondary school mathematics. Within the South African Curriculum and Assessment Policy Statement (CAPS), learners are formally introduced to circle geometry in the Senior Phase, with increasing emphasis on cyclic quadrilateral and tangent theorems in Grades 10–12. Despite this curricular importance, persistent underperformance in Euclidean geometry has been widely reported in National Senior Certificate diagnostic reports, particularly in questions requiring multi-step reasoning and formal proof.

Research consistently shows that many learners approach circle geometry procedurally, relying on memorisation of theorem statements rather than developing relational understanding of geometric structures. As a result, learners often struggle to interpret geometric configurations, coordinate multiple theorems within a single proof, and justify their reasoning deductively. Teacher exacerbates these challenges-centred instructional approaches that prioritise demonstration and rehearsal over exploration, conjecture, and sense-making.

The integration of dynamic geometry software has been proposed as one way to address these longstanding challenges. GeoGebra, in particular, allows learners to construct and manipulate geometric figures dynamically, providing immediate visual and numerical feedback. Through dragging, measuring, and experimentation, learners can explore invariant geometric relationships that are hidden in static diagrams. International and South African studies suggest that such environments can support conceptual understanding, learner engagement, and exploratory reasoning when embedded within appropriate pedagogical frameworks.

However, while the benefits of GeoGebra for visualisation and engagement are well documented, less is known about how learners appropriate the software as a tool for mathematical reasoning, particularly in relation to the development of deductive understanding. Moreover, research indicates that improved visual understanding does not automatically translate into competence in formal proof, raising important questions about the limits and possibilities of dynamic geometry environments.

Guided by constructivist learning theory and the framework of instrumental genesis, this study investigates the use of GeoGebra as an instructional instrument in teaching cyclic quadrilateral and tangent theorems to Grade 11 learners in a South African secondary school. The study examines how learners engage with geometric concepts through dynamic exploration, how the teacher integrates GeoGebra into classroom practice, and how learners begin to appropriate the software as a reasoning instrument. In doing so, the study seeks to contribute to current debates on technology-enhanced geometry instruction by offering empirically grounded insight into both the affordances and limitations of GeoGebra in supporting conceptual understanding and deductive reasoning.

While a growing body of research documents the benefits of dynamic geometry software for learner engagement and visualisation, fewer studies have examined how learners appropriate such software as a reasoning instrument within classroom practice. In particular, there remains limited empirical evidence from South African secondary school contexts that traces how learners move from exploratory interaction with dynamic representations toward more structured geometric reasoning. This study addresses this gap by examining how Grade 11 learners begin to transform GeoGebra from a technological artefact into a cognitive instrument for reasoning about cyclic quadrilateral and tangent theorems (Mudaly and Fletcher, 2022). By foregrounding learners' evolving use of the software, rather than achievement outcomes alone, the study offers insight into both the potential and the limits of GeoGebra for supporting conceptual understanding and deductive development in Euclidean geometry.

Literature Review & Theoretical Framework

Challenges in Learning Circle Geometry

Euclidean geometry, and circle geometry in particular, remains a persistent challenge for secondary school learners. Research in the South African context highlights learners' difficulties with interpreting geometric configurations, coordinating multiple theorems, and constructing logical proofs. These difficulties are frequently attributed to instructional practices that emphasise memorisation of theorem statements rather than conceptual understanding and reasoning.

Skemp's distinction between instrumental and relational understanding remains useful in explaining learners' performance in geometry. Many learners demonstrate instrumental competence, recalling theorem statements without understanding why these relationships hold or how they can be applied flexibly. This gap is especially evident in circle geometry, where successful problem-solving often requires integrating several geometric properties into a coherent deductive argument.

Dynamic Geometry Software and Conceptual Understanding

Dynamic geometry environments provide learners with opportunities to explore geometric relationships through direct manipulation. GeoGebra integrates geometric construction, measurement, and algebraic representation within a single interactive platform, enabling learners to investigate invariant relationships through dragging and experimentation.

Previous studies report that learners using GeoGebra show improved conceptual understanding, greater confidence, and increased engagement in geometry lessons. The ability to coordinate visual and numerical representations supports sense-making and conjecture formation, key components of mathematical understanding. However, the literature also cautions that dynamic exploration alone is insufficient for developing formal proof competence, highlighting the need for deliberate pedagogical scaffolding.

Instrumental Genesis and Technology-Mediated Learning

The framework of instrumental genesis provides a powerful lens for examining how learners transform technological artefacts into instruments for thinking. According to Rabardel, an artefact becomes an instrument only when users develop cognitive schemes that integrate tool use with conceptual reasoning.

Instrumental genesis involves two intertwined processes: instrumentalisation, in which learners adapt the artefact to suit their needs, and instrumentation, in which the artefact shapes learners' reasoning strategies. In dynamic geometry environments, learners' use of dragging, measurement, and construction tools can evolve from procedural actions into instrumented techniques that support mathematical reasoning. The teacher plays a crucial role in supporting this process through task design and instructional orchestration (Sinclair et al. 2016).

Within this framework, the focus is not simply on whether learners use GeoGebra, but on how their use of the software changes over time. Instrumental genesis provides a means of examining this development by distinguishing between learners' operational interaction with the artefact and the emergence of instrumented techniques that support mathematical reasoning. This distinction is particularly useful in geometry, where visual exploration can remain superficial unless learners begin to integrate tool use with conceptual justification.

Constructivism and Geometric Reasoning

Constructivist learning theory emphasises that learners actively construct knowledge through interaction with tasks, tools, and peers. In geometry, understanding develops through cycles of exploration, conjecture, justification, and reflection. Dynamic geometry environments align well with constructivist principles by providing opportunities for experimentation and immediate feedback. However, effective constructivist teaching requires carefully designed tasks that connect exploratory activity to formal mathematical reasoning.

Methodology

Research Design

This study employed a qualitative case study research design to explore how GeoGebra serves as an instructional tool to enhance Grade 11 learners' understanding of cyclic quadrilaterals and the tangent theorems. A case study approach was chosen because it enables a detailed investigation of a bounded system in its real-life context (Yin, 2018). The design enabled the researcher to observe classroom interactions, learner engagement, and the implementation of GeoGebra-supported teaching as they naturally occurred.

The study was located within a constructivist research paradigm, which assumes that knowledge is actively constructed through interaction with tools, tasks, and social partners (Vygotsky, 1978). In addition, the study was theoretically informed by the framework of instrumental genesis (Rabardel, 1995; Trouche, 2004), which guided the investigation into how learners transformed GeoGebra from a technological artefact into a cognitive instrument for geometric reasoning.

The case study design was particularly appropriate for this study because the focus was not on measuring GeoGebra's effectiveness in a comparative or experimental sense, but on understanding how learners engaged with the software within a specific instructional context. This design allowed for close attention to classroom interaction, learner activity, and the evolving use of GeoGebra during instruction, which would not have been accessible through large-scale or purely quantitative approaches.

Research Context

The study was carried out at a public secondary school in the Metro East Education District of the Western Cape, South Africa. The school caters to learners from lower- to middle-income backgrounds and reflects many urban public schools in the province. It had access to a computer laboratory equipped with desktop computers, allowing the implementation of the GeoGebra-supported instructional intervention. Teaching Euclidean geometry at the school follows the CAPS Mathematics curriculum as outlined by the Department of Basic Education (DBE, 2011). Before the intervention, students had been introduced to basic properties of circles but had limited experience with dynamic geometry software.

Participants

Participants in the study comprised:

- One Grade 11 mathematics teacher implemented the GeoGebra-supported lessons.
- Thirty-nine (39) Grade 11 learners from one intact mathematics class.

A purposive sampling strategy was used to select the school, teacher, and class because they met the study's key criteria: availability of ICT infrastructure, willingness to participate, and alignment with the research objectives (Creswell, 2014). The learners comprised a mixed-ability class, reflecting the range of achievement levels typically found in South African secondary-school mathematics classrooms.

The Instructional Intervention

The GeoGebra-supported instructional intervention was implemented over a three-week period and was structured into three progressive instructional cycles. Each cycle was designed to support learners' gradual appropriation of GeoGebra as a learning instrument and to scaffold their understanding of circle geometry concepts.

Cycle 1: Introduction to GeoGebra

The first cycle focused on familiarising learners with the GeoGebra interface and tools. Learners were introduced to fundamental construction tools, including points, lines, circles, angle measurement, and the drag function. The purpose of this cycle was to reduce technical barriers to learning and initiate the process of instrumental genesis by enabling learners to develop basic operational control of the software (Rabardel, 1995).

Cycle 2: Guided Exploration of Cyclic Quadrilaterals and Tangents

The second cycle centred on guided GeoGebra tasks involving the construction of cyclic quadrilaterals and tangent lines. Learners constructed geometric configurations and used measurement and dragging tools to explore relationships between opposite angles, exterior angles, and tangent-chord angles. Tasks were designed to encourage the formulation of conjectures, visual verification, and collaborative discussion, consistent with constructivist learning principles (Vygotsky, 1978).

Cycle 3: Independent Exploration and Informal Proof

The third cycle emphasised learner autonomy. Learners independently solved more complex circle-geometry problems using GeoGebra. They were encouraged to test conjectures, construct auxiliary lines, and attempt informal explanations of their reasoning. This cycle aimed to consolidate conceptual understanding and support learners' transition toward deductive reasoning, in line with the van Hiele model (van Hiele, 1986).

Data Collection Methods

Multiple qualitative data sources were used to ensure data triangulation and to strengthen the credibility of the findings (Creswell, 2014).

Classroom Observations

Non-participant classroom observations were conducted during all three instructional cycles. The observations focused on learner engagement, use of GeoGebra tools, interaction patterns, and the teacher's instructional strategies. Detailed field notes were recorded to document learners' behaviours, verbal interactions, and emerging learning processes.

Semi-Structured Interviews

Semi-structured interviews were conducted with the participating teacher and a purposively selected group of learners after the intervention. The interviews explored participants' experiences with GeoGebra, their perceived impact on understanding circle geometry, and the challenges encountered during the lessons. All interviews were audio-recorded and transcribed verbatim for analysis.

Learner Survey

A post-intervention learner survey was administered to 20 learners, representing approximately 50% of the class. The survey included Likert-scale items and open-ended questions to capture learners' perceptions of GeoGebra's usefulness, ease of use, confidence in geometry, and its impact on conceptual understanding. The survey data were used to complement the qualitative findings.

GeoGebra Artefacts

Learners' GeoGebra constructions, screenshots, and saved files were collected during the intervention. These digital artefacts provided visual evidence of learners' reasoning processes and were used to support the interpretation of instrumental genesis and conceptual development.

Data Analysis

Data analysis followed a thematic analysis approach (Braun & Clarke, 2006). Interview transcripts, observation notes, survey responses, and GeoGebra artefacts were coded inductively to identify recurring patterns and themes.

The framework of instrumental genesis (Rabardel, 1995; Trouche, 2004) informed the analysis by guiding attention to:

- Evidence of instrumentalisation, where learners adapted the software to suit their learning needs.
- Evidence of instrumentation in which the software shaped learners' reasoning strategies.

Constructivist learning theory (Vygotsky, 1978) and the van Hiele model (van Hiele, 1986) further informed the interpretation of learners' conceptual development and progression in geometric reasoning.

Trustworthiness of the Study

To ensure the trustworthiness of the study, the following strategies were employed (Lincoln & Guba, 1985):

- Credibility was enhanced through data triangulation across observations, interviews, surveys, and artefacts.

- Transferability was supported through rich, thick descriptions of the research context and participants.

- Dependability was established through the maintenance of a clear audit trail of data collection and analysis procedures.

- Confirmability was strengthened through the use of multiple data sources and the transparent documentation of analytical decisions.

Ethical Considerations

Ethical clearance for the study was obtained from the University of the Western Cape Research Ethics Committee under reference number HS23/7/41. The Western Cape Education Department and the participating school granted permission to conduct the research.

Informed consent was obtained from:

- The participating teacher,
- All participating learners,
- Learners' parents or legal guardians.

Participants were assured of anonymity and confidentiality. Pseudonyms were used in all reporting, and all data were stored securely on password-protected digital devices. Participation was voluntary, and participants were informed of their right to withdraw from the study at any stage without penalty.

Findings

Overview of the Findings

The findings are organised according to four central themes that emerged from the thematic analysis of classroom observations, learner and teacher interviews, learner survey responses, and GeoGebra artefacts. These themes are:

1. Development of visualisation and conceptual understanding
2. Learner engagement and collaborative learning
3. Evidence of instrumental genesis
4. Challenges in the development of formal proof

These themes collectively reflect how learners engaged with GeoGebra and how their understanding of cyclic quadrilateral and tangent theorems developed during the intervention.

Development of Visualisation and Conceptual Understanding

A significant finding of the study was the marked improvement in learners' ability to visualise and conceptually interpret geometric relationships in circle geometry. Prior to the intervention, many learners relied predominantly on memorised theorem statements and struggled to interpret static textbook diagrams. During the GeoGebra-supported lessons, learners demonstrated increasing ability to interpret dynamic geometric configurations and to identify invariant relationships through manipulation.

Using the dragging tool, learners observed that the opposite angles in a cyclic quadrilateral remained supplementary, regardless of how the quadrilateral was distorted. This dynamic verification supported a shift from rote recall toward relational understanding, as described by Skemp (1976). Learners frequently articulated statements such as:

No matter how we move the points, the opposite angles always add up to 180°.

Similarly, when exploring tangent theorems, learners used measurement tools to verify that the angle between the tangent and chord was equal to the angle in the opposite arc. These observations confirmed the value of dynamic visualisation in supporting conceptual learning, as emphasised by Duval (1999) and Jones et al. (2007).

The immediate numerical feedback provided by GeoGebra's measurement tools further strengthened the link between visual and symbolic representations. This coordination of representations is considered essential for the development of robust conceptual understanding in mathematics (Duval, 1999).

Learner Engagement and Collaborative Learning

Classroom observations revealed a notable increase in learner engagement throughout the three instructional cycles. Learners demonstrated high levels of participation, voluntary exploration, and sustained attention during GeoGebra-based activities. This finding aligns with previous studies indicating that dynamic geometry environments promote learner motivation and active engagement (Diković, 2009; Mthethwa, 2015).

Learners became increasingly willing to experiment with geometric constructions because of the low risk in the digital environment. The ability to undo actions and test multiple configurations without permanent consequences encouraged learners to take intellectual risks and engage more deeply with the content. This form of exploratory engagement aligns with the principles of constructivist learning, in which learners actively construct meaning through interaction and experimentation (Vygotsky, 1978).

Collaborative learning was also strongly evident. Learners worked in small groups, shared conjectures, debated geometric relationships, and assisted one another in troubleshooting construction errors. Through peer discussion, learners negotiated meaning and refined their understanding of geometric properties. These interactions are consistent with social constructivist perspectives, which stress the role of dialogue and social mediation in learning (Vygotsky, 1978).

Evidence of Instrumental Genesis

A central finding of the study was the clear emergence of instrumental genesis, as learners progressively transformed GeoGebra from a digital artefact into an instrument for geometric reasoning (Rabardel, 1995; Trouche, 2004).

Instrumentalisation

Instrumentalisation was evident in how learners adapted GeoGebra's tools to meet their cognitive needs. Learners developed personal strategies for:

- Using colour coding to differentiate among angles and arcs,
- Applying the measurement tool to verify relationships,
- Constructing auxiliary lines to support reasoning,
- Using the undo function strategically to test multiple cases.

These actions indicate that learners were not merely following procedural instructions but were actively shaping the artefact for their own learning purposes, as described by Rabardel and Béguin (2005).

Instrumentation

Instrumentation was observed as the dynamic features of GeoGebra began to influence learners' reasoning strategies. Instead of immediately recalling theorem statements, learners increasingly relied on exploration, conjecture, and visual verification before attempting justification. This shift reflects how the tool began to shape learners' cognitive activity, a key feature of instrumental genesis (Trouche, 2004).

Learners frequently used dragging to test invariance and measurement tools to confirm numerical consistency. These practices demonstrate the development of instrumented techniques, similar to those identified by Bozkurt and Uygan (2020) in their study of dynamic geometry environments.

Learner Survey Findings

Survey data further supported the qualitative findings. The majority of learners indicated that GeoGebra improved their understanding of circle geometry and made learning more enjoyable.

Key trends from the survey included:

- Most learners agreed that GeoGebra helped them "see" how the theorems work rather than merely memorising formulas.
- Learners reported greater confidence when attempting circle geometry problems.
- Many learners preferred learning geometry with GeoGebra rather than through traditional textbook-based instruction.

These findings are consistent with the results reported by Chimuka and Ogbonnaya (2015) and Zengin et al. (2012), who found that learners exposed to GeoGebra demonstrated improved conceptual understanding and more positive attitudes toward mathematics.

Teacher Reflections on the Use of GeoGebra

The participating teacher reported that GeoGebra significantly transformed classroom interaction. The teacher observed that learners were more engaged, asked more meaningful questions, and displayed greater independence in exploring geometric relationships. The teacher further noted that GeoGebra facilitated the identification of learner misconceptions because errors became immediately visible through incorrect constructions or inconsistent measurements (Mhlolo, 2021

However, the teacher also indicated that successful integration of GeoGebra required careful lesson planning, time allocation, and confidence in using the software. Without adequate preparation and pedagogical support, the teacher believed that GeoGebra could easily be reduced to a demonstration tool rather than a learner-centred exploration tool.

Challenges in the Development of Formal Proof

Despite substantial gains in visualisation and conceptual understanding, the findings revealed that many learners continued to struggle to translate their visual observations into formal deductive proofs. During the third instructional cycle, several learners relied heavily on empirical evidence obtained through dragging and measurement rather than constructing logically coherent arguments.

This finding echoes the concerns raised by Harel and Sowder (2007) that learners often struggle to move from empirical reasoning to formal proof when they lack sustained exposure to structured deductive practices. Although GeoGebra supported learners at the visual and informal deduction levels of the van Hiele model, progression to the formal deduction level required additional scaffolding (van Hiele, 1986).

Conclusion

The findings of the study indicate that:

- GeoGebra significantly enhanced learners' visualisation and conceptual understanding of cyclic quadrilateral and tangent theorems.
- Learner engagement and collaborative learning increased markedly during the intervention.
- Clear evidence of instrumental genesis was observed as learners developed instrumented techniques and increasingly used GeoGebra as a reasoning tool.
- Challenges remained in learners' ability to translate visual reasoning into formal deductive proof.

These findings highlight both the strengths and limitations of GeoGebra as an instructional tool for teaching and learning circle geometry.

Discussion

Contribution of the Study

The discussion that follows moves beyond a description of the findings to consider their significance in relation to existing research on dynamic geometry environments, instrumental genesis, and geometric reasoning. The intention is not to generalise beyond the case studied, but to situate the findings within broader debates about the role of technology in supporting conceptual understanding and deductive reasoning in secondary school mathematics (Adler and Pillay, 2024).

This study contributes to mathematics education research in three important ways. First, it provides empirical evidence of how instrumental genesis unfolds in a South African Grade 11 geometry classroom, extending existing research on GeoGebra beyond learner perceptions and achievement outcomes to focus on the development of instrumented reasoning practices. Second, the study demonstrates that while dynamic geometry environments significantly enhance visualisation and conceptual understanding, these gains do not automatically translate into formal deductive proof competence, thereby complicating optimistic claims about technology-enhanced learning. Third, by foregrounding the role of teacher orchestration and task design, the study offers practical insights for integrating GeoGebra as a reasoning instrument rather than as a demonstration or drawing tool. These contributions have direct implications for geometry teaching, curriculum implementation, and teacher professional development in the South African context.

GeoGebra as a Mediating Instrument for Conceptual Understanding

The findings of this study demonstrate that GeoGebra served as a powerful mediating tool in supporting learners' conceptual understanding of the cyclic quadrilateral and tangent theorems. Through dynamic manipulation and real-time visual feedback, learners could observe invariant geometric relationships, test conjectures, and refine their reasoning. This aligns with the existing literature, which argues that dynamic geometry environments promote relational understanding rather than rote procedural recall (Diković, 2009; Zengin et al., 2012).

Learners' frequent use of dragging to verify that the opposite angles in a cyclic quadrilateral remain supplementary across all configurations reflects a shift from an instrumental to a relational understanding, as described by Skemp (1976). Instead of memorising theorem statements in isolation, learners could see how these relationships emerged and persisted through continuous transformation. This experiential form of learning enabled learners to develop deeper insight into the underlying structure of circle geometry.

Furthermore, GeoGebra's capacity to link numerical and visual representations enhanced learners' coordination of multiple representations. Duval (1999) argues that conceptual understanding in mathematics requires learners to move flexibly between visual, symbolic, and numerical registers. The present study confirms that learners' ability to interpret angle measures alongside dynamic geometric figures supported a more coherent and integrated understanding.

However, while GeoGebra effectively supported conceptual understanding, the findings also revealed that it did not automatically lead to successful formal proof construction. Many learners continued to rely on empirical verification through dragging and measurement rather than developing logically structured deductive arguments. This confirms the caution noted in the literature that visual evidence alone is insufficient for establishing formal mathematical proof (Harel & Sowder, 2007).

Learner Engagement and Constructivist Learning

The increased learner engagement observed during the intervention reflects the strong alignment between GeoGebra-supported instruction and constructivist learning principles. Learners were actively involved in constructing their own understanding through exploration, conjecture, and collaborative problem-solving, rather than passively receiving information from the teacher. This finding is consistent with the constructivist view that learning occurs most effectively when learners are cognitively and socially engaged in meaningful tasks (Vygotsky, 1978).

The low-risk nature of the digital environment contributed to a shift in learner disposition toward mathematics. The ability to undo actions and test multiple cases encouraged learners to take intellectual risks without fear of permanent error. This created a classroom culture characterised by curiosity, experimentation, and persistence. Such an environment supports the development of what Vygotsky (1978) describes as mediated learning, where tools and social interaction jointly shape cognitive development.

Collaborative learning was also a prominent feature of the GeoGebra-supported lessons. Learners shared constructions, debated geometric relationships, and supported one another in resolving conceptual and technical difficulties. These interactions enabled learners to negotiate meaning collectively, refine their reasoning through dialogue, and co-construct understanding. Social interaction thus served as a critical mediating process in the development of learners' geometric thinking.

Instrumental Genesis and the Development of Geometric Reasoning

A central theoretical contribution of this study lies in the empirical demonstration of instrumental genesis in a South African secondary school context. The findings provide clear evidence of both instrumentalisation and instrumentation as learners appropriated GeoGebra for geometric reasoning.

Through instrumentalisation, learners adapted GeoGebra's features to suit their cognitive needs. They developed personalised techniques such as colour-coding angles, strategic use of the measurement tool, and the construction of auxiliary lines to support reasoning. These adaptations indicate that learners actively shaped the artefact in response to their learning goals, consistent with Rabardel’s (1995) conceptualisation of instrumentalisation.

Instrumentation was observed as the dynamic properties of GeoGebra began to shape learners’ reasoning strategies. Instead of relying solely on memorised procedures, learners increasingly engaged in visual exploration, conjecture formation, and empirical verification before attempting justification. This reflects how the tool itself began to shape learners’ thinking and approach to geometric problems (Trouche, 2004).

The emergence of instrumented techniques such as dragging to test invariance and using perpendicular radii to construct tangents mirrors the findings of Bozkurt and Uygan (2020), who demonstrated that sustained engagement with dynamic geometry software leads to the integration of technological tools into learners’ mathematical thinking. Importantly, this study also confirms the critical role of the teacher in supporting instrumental genesis through appropriate task design and instructional orchestration, as emphasised by Drijvers et al. (2010).

GeoGebra and Progression Through van Hiele Levels

The findings suggest that GeoGebra effectively supported learners’ progression through the lower and intermediate levels of the van Hiele model of geometric thinking. Learners demonstrated marked improvement at the visualisation level, becoming more adept at recognising geometric shapes and configurations, and at the analysis level, where they began to identify and describe relationships among geometric properties (van Hiele, 1986).

At the informal deduction level, learners showed increased ability to formulate conjectures and attempt explanations based on observed relationships. The dynamic nature of GeoGebra enabled learners to explore “what happens if” scenarios, test hypotheses, and refine their reasoning through repeated cycles of exploration and reflection. This process aligns with the van Hiele model’s emphasis on activity-based learning as a catalyst for progression between levels.

However, the study also indicates that many learners remained challenged at the formal deduction level, where learners are expected to construct logically coherent proofs. While GeoGebra supported exploration and informal reasoning, constructing formal proofs required additional scaffolding and explicit instruction in deductive methods. This confirms previous findings that technological tools, while powerful, cannot replace the need for structured proof instruction (Harel & Sowder, 2007).

Implications for the Teaching of Circle Geometry in South Africa

The findings of this study have important implications for the teaching of circle geometry within the South African CAPS curriculum. Persistent underperformance in Euclidean geometry suggests a need to move beyond traditional pedagogies toward more learner-centred, exploratory approaches that emphasise conceptual understanding (Mudaly & Fletcher, 2022)

GeoGebra is a practical, accessible tool for achieving this shift. As a free and open-source software platform, it has significant potential for adoption in both well-resourced and resource-constrained school contexts. When integrated within a constructivist pedagogical framework and supported by appropriate teacher professional development, GeoGebra can enhance learners' engagement, conceptual understanding, and mathematical sense-making.

However, the findings also highlight that meaningful integration of GeoGebra requires careful attention to pedagogical design. Without deliberate scaffolding and instructional guidance, learners may remain at a superficial level of tool use and fail to develop formal deductive competence. Teachers, therefore, play a central role in orchestrating learning experiences that link dynamic exploration to formal mathematical reasoning (Ng, Sinclair and Patahuddin 2023)

Consolidated discussion

The discussion reveals that:

- GeoGebra functioned as a powerful mediating instrument for conceptual understanding in circle geometry.

- Learner engagement and collaboration increased substantially under constructivist instructional conditions.

- Clear evidence of instrumental genesis was observed as learners appropriated GeoGebra as a reasoning tool.

- While visual and conceptual understanding improved markedly, challenges persisted in developing formal deductive proofs.

These findings affirm that GeoGebra is most effective when integrated as a thinking instrument within a coherent pedagogical framework, rather than as a stand-alone technological add-on to traditional instruction.

Conclusion

This study investigated how GeoGebra functions as an instructional tool to facilitate Grade 11 learners' understanding of the cyclic quadrilateral and tangent theorems in circle geometry. The findings demonstrate that integrating GeoGebra within a constructivist instructional framework significantly enhanced learners' visualisation, conceptual understanding, and engagement with Euclidean geometry. Learners moved beyond rote memorisation of theorem statements toward relational understanding, supported by dynamic manipulation, real-time feedback, and collaborative exploration.

The study further confirmed that GeoGebra did not merely function as a digital drawing tool, but became a cognitive instrument through the process of instrumental genesis (Rabardel, 1995; Trouche, 2004). Learners developed instrumented techniques such as dragging to test invariance, constructing perpendicular radii to form tangents, and strategically using measurement tools to verify relationships. These practices reflect a meaningful transformation in how learners reasoned about geometric relationships.

However, while learners' visual and conceptual understanding improved considerably, the findings also revealed that many learners continued to struggle to translate their empirical observations into formal deductive proofs. This highlights the important distinction between developing intuitive understanding and achieving full deductive competence (Harel & Sowder, 2007; van Hiele, 1986).

Overall, the study provides strong evidence that GeoGebra, when integrated within a carefully designed pedagogical framework, is an effective instructional instrument for enhancing the teaching and learning of circle geometry in the South African secondary school context.

Taken together, the findings suggest that the educational value of GeoGebra lies not only in its capacity to enhance visualisation but also in its pedagogical integration to support learners' reasoning. When learners are guided to use dynamic representations as tools for conjecture, testing, and reflection, GeoGebra can contribute meaningfully to conceptual understanding. However, the study also underscores that developing formal proof requires sustained instructional attention beyond dynamic exploration alone.

Implications for Teaching and Learning

The findings of this study have important implications for classroom practice, curriculum implementation, and teacher professional development.

Implications for Teachers

Teachers are encouraged to move beyond predominantly teacher-centred approaches toward pedagogies that emphasise exploration, conjecture, justification, and learner interaction. GeoGebra provides a practical tool for implementing such pedagogical shifts by enabling learners to explore geometric relationships dynamically rather than relying solely on static diagrams.

However, meaningful integration requires more than technical competence. Teachers need to:

- Design tasks that deliberately link visual exploration to formal reasoning,
- Provide explicit scaffolding for proof development,
- Orchestrate classroom discussion to support conceptual and deductive growth (Drijvers et al., 2010),
- Guide learners in moving from empirical verification to logical justification.

Professional development programmes should therefore prioritise both technological and pedagogical training in dynamic geometry environments.

Implications for Learners

Learners benefit from environments that promote active involvement, experimentation, and collaboration. GeoGebra creates a low-risk environment that encourages learners to test ideas, make mistakes, and refine their reasoning without fear of permanent error. This supports the development of confidence, mathematical curiosity, and persistence in problem-solving (Vygotsky, 1978).

Importantly, learners must also be guided to understand that visual evidence alone is insufficient as proof. They need sustained exposure to deductive reasoning practices that connect empirical observation to formal logical argumentation.

Curriculum and Policy Implications

At the curriculum and policy levels, the findings support a stronger, more explicit integration of dynamic digital tools within the CAPS Mathematics curriculum. While the use of technology is encouraged in principle (DBE, 2011), practical guidance on how tools such as GeoGebra can support specific geometry topics remains limited.

Clear curriculum guidance, together with adequate school-level infrastructure and teacher training support, could significantly enhance the quality of Euclidean geometry teaching nationally.

Limitations of the Study

Several limitations of the study must be acknowledged.

First, the study was conducted within a single school and one Grade 11 class, which limits the generalisability of the findings. While the qualitative case study design enabled in-depth exploration, the results cannot be generalised to all South African school contexts.

Second, the duration of the intervention was relatively short (three weeks). Although meaningful learning gains were observed, the study could not establish whether these gains were sustained over time or translated into long-term improvement in formal proof competence.

Third, the intervention involved one teacher, whose pedagogical style, technological confidence, and enthusiasm may have influenced the outcomes. Different results may emerge among teachers with varying levels of experience and technological proficiency. Finally, although the learner survey provided useful, supportive evidence, the study remained primarily qualitative. A mixed-methods or experimental design could provide stronger statistical evidence of learning gains attributable to GeoGebra.

Recommendations for Future Research

Future research could extend the findings of this study in several important directions:

1. Longitudinal studies examining whether GeoGebra-supported conceptual gains are sustained into Grade 12 and beyond, particularly in relation to formal proof competence.

2. Large-scale comparative studies comparing GeoGebra-supported instruction with traditional teaching approaches across different provinces and school contexts.

3. Experimental or quasi-experimental studies to measure the impact of GeoGebra quantitatively on learner achievement in circle geometry.

4. Teacher-focused studies exploring how instrumental orchestration practices develop and how they influence learner outcomes.

5. Equity-focused research investigating how GeoGebra integration impacts learners in under-resourced rural and township schools.

Such research would further strengthen the empirical foundation for systematic integration of dynamic geometry software in South African mathematics education.

Final Remarks

This study affirms that GeoGebra is not merely a technological enhancement to traditional instruction but a powerful mediating instrument capable of transforming how learners engage with geometric reasoning. When aligned with constructivist pedagogy and supported by deliberate instructional design, GeoGebra enables learners to see, test, and reason about geometric relationships in ways not possible with static representations alone.

In a context where Euclidean geometry remains a persistent barrier to learner success, the meaningful integration of dynamic geometry software, such as GeoGebra, offers a viable and impactful pathway to improve mathematical understanding, learner engagement, and overall performance in secondary school mathematics.

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Conflict of Interest

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